

Three ways in which logic might be normative*

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January 27, 2015

1 Introduction

Logic, the tradition has it, is normative. As Frege put it, logic ‘prescribe[s] universally how one ought to think if one is to think at all’ (Frege 1893/1903/2009, p. xv). On the face of it our intuitions accord with the tradition. It’s a bad thing, we tend to think, to be inconsistent. Similarly, we criticize people for failing to appreciate at least the more obvious logical consequences of their beliefs when these are in question. Call the thesis that logic has a normative role to play in reasoning—i.e. in the ways we go about forming and revising our doxastic attitudes or perhaps our evaluations thereof—the *Normativity Thesis* (NT).

NT has come under heavy fire. Gilbert Harman (and a number of philosophers in his wake) have argued tirelessly that there is no ‘significant way in which *logic* is specially relevant to reasoning’ (Harman 1986, p. 20, the emphasis is Harman’s).¹ In recent years, others have sought to defend the notion that logic has a normative role to play in our cognitive economy against Harman’s attacks.² I ultimately side with NT’s defenders. However, I believe that before we can hope to make any significant headway on the question of NT’s tenability, we must get clearer on what it is that we are asking when we are asking after the normativity of logic.

*I am grateful to Tim Button, David Christensen, Matti Eklund, Hartry Field, Branden Fitelson, Hannah Ginsborg, Anandi Hattiangadi, Rosanna Keefe, Hannes Leitgeb, John MacFarlane and Robbie Williams for helpful discussions and comments on earlier versions of this paper.

¹Harman’s criticisms are central to Harman’s work in the theoretical domain. They go back at least as far as his (Harman 1973) (see also (Harman 1984, 2002; Harman and Kulkarni 2007; Harman 2009, 2010) among others). Arguably, these ideas have their roots in Frege’s anti-psychologism. They also appear to have been anticipated by Carnap (1962, §11). More recently, similar ideas have been defended in (Goldman 1986).

²Field (2009a, Forthcoming), MacFarlane (2004), Milne (2009).

Put another way: In order to properly assess NT’s philosophical viability, we must first clarify and indeed disambiguate NT. This paper is an attempt to undertake this clarificatory task. Its central claim is that there are in fact several importantly different ways in which logic might be thought to be normative. In a first step, I distinguish three distinct normative roles that logic could conceivably play: Logical norms might provide *directives* that guide agents engaged in doxastic deliberation; alternatively, logical norms might serve to make *evaluations*, setting standards by which to assess an agent’s doxastic state for its logical cogency; or, finally, they might play the role of *appraisals* by which we criticize, praise or otherwise hold accountable an agent for her doxastic conduct.

This conceptual ground-clearing, I go on to show, allows us to disambiguate and to variously reformulate NT. It is therefore indispensable for a proper understanding of the question of the normativity of logic, allowing us to replace the original question with three well-defined and hence more tractable questions: “Is logic normative in the sense of providing directives/evaluations/appraisals?” In a second step, I show that a failure fully to appreciate the proposed three-fold distinction is at the source of a number of shortcomings and confusions plaguing the current debate. Finally, I address the question of NT’s status once disambiguated in this way. In particular, I argue that, logic can be said to be said to be normative both in the sense of furnishing directives and in the sense of setting evaluative standards.

I proceed as follows. Section 2 summarizes Harman’s criticisms of NT. In particular, I identify four key objections the avoidance of which will later serve as desiderata against which positive accounts of NT may be measured. Section 3 introduces the notion of a *bridge principle*. Bridge principles are general principles that articulate the ways in which a valid argument (or our attitudes towards such an argument) normatively constrains specified doxastic attitudes. I introduce and further develop John MacFarlane’s (2004) helpful taxonomy of bridge principles. In section 4, I distinguish the three types of normative roles logic might be thought to play in our epistemic lives. I explain how these distinctions affect the ways in which bridge principles are to be interpreted and assessed. An important upshot is that because key contributions to the debate surrounding NT—contributions by Harman, MacFarlane and Hartry Field—largely fail to appreciate our three-fold distinction between different types of normative roles, they are led to talk past one another (at least in part): MacFarlane’s and Field’s proposals, despite their considerable independent merit, interpret NT in ways different Harman’s interpretation and therefore fail to engage with Harman on his own terms. In the final sections, I address the question whether logic can be said to be normative in the first two proposed senses: ‘Does logic provide directives to guide doxastic deliberation?’ and ‘Does logic set evaluative standards for assessing others?’

2 Harman’s skeptical challenge

At the root of Harman’s criticism lies a diagnosis: The traditional conception whereby logic occupies a normative role in our cognitive economy rests upon a ‘category mistake’—the mistake of conflating principles of deductive logic with what Harman calls ‘a theory of reasoning’. Yet the two enterprises—formulating a deductive logic and formulating a normative theory of reasoning—are fundamentally different according to Harman. A theory of reasoning is a theory of how ordinary agents ought to reason: How we should go about managing our beliefs.³ Thus, the subject matter of a theory of reasoning are the dynamic ‘psychological events or processes’ by which we form, revise or retain beliefs. The theory seeks to formulate general guidelines as to which mental actions (judgments and inferences) to perform in which circumstances and which doxastic attitudes (belief, disbelief, suspension) to form, to retain or to abandon (Harman 2009, p. 333). In contrast, ‘the sort of implication and argument studied in deductive logic have to do with [static, non-psychological] relations among propositions’. Consequently, ‘logical principles are not directly rules of *belief revision*. They are not particularly about belief [or the other mental states and acts that constitute reasoning] at all’ (Harman 1984, p. 107).

This category mistake is enshrined in our perniciously misleading use of the label ‘rule of inference’ to designate deductive rules (Modus Ponens, say). Deductive rules state (typically elementary) logical implications between certain types of propositions based on their logical form. Though they may be seen to state permissible deductive steps within a specified formal calculus, they bear no straightforward relation to the mental act of inferring. Reasoning is fundamentally different from, and by no means reducible to, the activity of carrying out derivations in a formal proof system. As far as Harman is concerned, ‘inference’ figures in ‘inference rule’ like ‘democratic’ does in ‘German Democratic Republic’ as—in the Quinian phrase—an ‘orthographic accident’. Similar comments apply to our talk of ‘deductive reasoning’.⁴

³Some maintain that doxastic states (and perhaps also mental acts like judging and inferring) cannot be said to be under the agent’s voluntary control and so are not responsive to reasons in ways they would need to be in order to be proper subjects of normative appraisal. See (Alston 1988) and (Williams 1976) for standard arguments against doxastic voluntarism. I will make no attempt to broach this question here. For current purposes I will simply assume that talk of epistemic norms can be made sense of roughly along the lines proposed by (Ryan 2003), (Steup 2000) and (Weatherson 2008).

⁴I will mostly set aside the case of practical reasoning here. It is important to note, however, that Harman believes there to be an analogous fallacy consisting, e.g. in the identification of a theory of practical reasoning with decision theory

The principles of mathematical decision theory are like principles of logic in being principles of consistency or coherence. So it is as much of a mistake to identify

Harman maintains that the differences in subject matter between deductive logics and theories of reasoning entrain differences in the character of the theories' principles and in their range of applications. Where logic deals in exception-less deductive rules, the principles of a theory of reasoning are, perforce, *defeasible*, applying only 'all things being equal' (Harman 1984, p. 109). Principles of reasoning, unlike logical principles, may be trumped by competing epistemic and practical considerations. Also, logical rules of implication or deduction 'apply to any subject matter at all' (Harman and Kulkarni 2007, p. 560), whereas the rules in a theory of reasoning are domain-specific, they are 'are specifically principles about a particular process, namely reasoning' (*idem*).

In short, Harman explains our intuitions to the effect that logic has a normative role to play in reasoning as follows: We conflate (or at least run together too closely) deductive logic and a theory of reasoning. Once disabused of this confusion, Harman maintains, it is not clear that logic should have any more relevance for reasoning than any other theory. Just as it behooves us to be informed about the physical sciences if we seek to reason responsibly about certain aspects of reality, it behooves to be conversant with the principles of logic. No privileged normative status sets logic apart from other bodies of knowledge.

So much for our characterization of the nature of Harman's skeptical challenge. There are a number of ways one might try to respond to Harman's challenge. One line of response might be to take issue with Harman's way of setting up the problem. We might question his initial diagnosis of the source of our NT-favoring intuitions. That is, we might reject his diagnosis of the provenance of these intuitions as stemming from a mistaken identification of deductive logic and theories of reasoning. It might be thought, for instance, that Harman is led to exaggerate the gulf between deductive logic and theories of reasoning as a result of a contestable—because overly narrow—conception of either logic or reasoning, or both.

As far as Harman's conception of logic is concerned, advocates of broadly logical accounts of belief revision (belief revision theories, non-monotonic logics, dynamic doxastic logic, etc.) may feel that Harman is driven to his skepticism by failing to take more sophisticated logical tools into account. In Harman's defense it should be said, that Harman is not unaware of such approaches. Rather he seems to judge them to be ill-conceived. As for my own view, I am certainly not opposed to such approaches—quite to the contrary. All the same, it is worth getting clear about the question of the normativity of logic in the basic terms in which Harman poses it. After all, any logical way of modeling belief (more or less tacitly) takes a stance

mathematical decision theory with the theory of practical rationality as it is to identify the theory of theoretical rationality with logic (Harman 2002, p. 175).

on the question of NT. To shed light on the status of NT, is thus to shed light on the philosophical foundations of these approaches.

Regarding Harman’s conception of reasoning, some philosophers—especially (though not only) externalists about epistemic justification of various stripes—are likely to find fault with Harman’s internalistic, broadly Cartesian first-personal conception of a theory reasoning. Eventually, when it comes to distinguishing different normative roles, I will have something to say about Harman’s conception of logic and especially about his conception of reasoning. For now, however, I want to consider the question of the normativity of logic in a way that involves taking Harman’s way of framing the question for granted.

Let it be granted for present purposes, therefore, that logic (narrowly construed) really is not a theory of reasoning.⁵ Even so, saying that deductive logic and theories of reasoning are distinct is one thing, affirming that there could not be an interesting normative connection between them is quite another. As a first stab at articulating such a connection, we might try the following: Theoretical reasoning aims to provide an accurate representation of the world. We accurately represent the world by having true, perhaps even knowledge-constituting doxastic attitudes. Our doxastic attitudes have contents—propositions, let us say—and these contents stand in certain logical relations with one another. Having an awareness of these logical relations would appear to be conducive to the end of having true beliefs and so is relevant to theoretical reasoning. In particular, the logical notions of entailment and consistency seem to be important to the theoretical project. If I believe truly, the truth of my belief will carry over to its logical consequences. Conversely, if my belief entails a falsehood it cannot be true. Similarly, if the set of propositions I believe (in general or in a particular domain) is inconsistent, they cannot possibly afford an accurate representation of the world; at least one of my beliefs must be false.

Notice that this simple reflection on the connection between logic and norms of reasoning leads us right back to the basic intuitions we invoked at the outset of this paper. Let us spell them out in the form of the following two principles. Let S be an agent and P a proposition.⁶

- Logical implication principle (IMP): If S ’s beliefs logically imply P , then S ought to believe that P .

⁵By ‘narrowly construed’ I mean logics that are not specifically designed to model belief revision like (dynamic) doxastic logic or theories of belief revision.

⁶The following principles are roughly those discussed by Harman. Harman’s formulations of IMP vary in that they employ different deontic modals (e.g., in his (Harman 2002, p. 172) Harman’s mentions ‘should’ and ‘may’, in his (Harman 1986, p. 11) he uses ‘can be a reason for’. We will return to the question of deontic modals in due course.

- Logical consistency principle (CON): S ought to avoid having logically inconsistent beliefs.

Notice that IMP and CON are distinct. IMP, in and of itself, says nothing to prohibit inconsistent or even contradictory beliefs, all it requires is that my beliefs be closed under logical consequence. CON, on the other hand, does not require that I believe the consequences of the propositions I believe, it merely demands that the set of propositions I believe be consistent.

3 The objections

IMP and CON thus are a first—somewhat naive, it turns out—attempt at pinning down the elusive normative relation between logic and norms of reasoning. They appear to be vulnerable to the following four objections most of which can be identified in Harman’s writings. It is worth spelling the objections out in some detail as they will later serve us as criteria of adequacy for future attempts at improving upon IMP and CON.

(1) Facts about validity do not imply (at least not in any straightforward way) corresponding rules of rational conduct in reasoning. Suppose I believe P and $P \supset Q$ (as well as Modus Ponens). The mere fact that I have these beliefs and that I recognize them to jointly entail Q does not normatively compel any particular attitude towards Q on my part. In particular, it is not the case in general that I ought to come to believe Q . After all, Q may be at odds with some of my deeply held beliefs, and so it would be unreasonable of me to slavishly follow Modus Ponens by, as it were, ‘adding Q to my belief box’. The rational course of ‘action’, rather, when Q is untenable is for me to relinquish my belief in at least one of my antecedent beliefs P and $P \supset Q$ on account of their unpalatable implications. Thus, logical principles do not invariably offer reliable guidance in deciding what to believe (at least, when the relation between logical principles and our practices of belief-formation are understood along the lines of IMP). In a sense, IMP offers too much instruction; it accords logic too strong a role in our belief forming processes. Let us therefore call this the *Too Much Instruction Objection*.

There is a closely related objection due to John Broome (2000, p. 85), which deserves separate mention. Broome observes that any proposition trivially entails itself. From IMP it follows that we *ought* to believe any proposition we in fact believe. But this seems patently false: I might hold any number of irresponsibly acquired beliefs. The fact that, by mere happenstance, I hold these beliefs, in no way implies that I ought to believe them. Call this variation of the Too Much Information Objection, *Broome’s Objection*.

(2) A related worry is that a reasoner who blindly heeds the precepts of logic acts irrationally because she espouses an infinite number of utterly useless beliefs. This is because any of the propositions I believe entails an infinite number of propositions that are of no interest to me whatsoever. Not only do I not care about, say, the disjunction ‘I am wearing blue socks or Elvis Presley was an alien’ entailed by my true belief that I am wearing blue socks, it would be positively irrational for me to squander my meager cognitive resources on inferring trivial implications of my beliefs that are of no value to my projects. Hence, in cases like these it again seems false to say that I ought to believe the logical consequences of my beliefs. Harman fittingly dubs the principle of reasoning in question *Principle of Clutter Avoidance*. Let us call the corresponding objection the *Objection From Clutter Avoidance*.

(3) Both principles—IMP and CON—place excessive demands on agents whose resources of time, computational power, stamina, etc., are limited. For instance, according to IMP anyone who believes the axioms of Peano arithmetic ought to believe every last of its theorems, even if its shortest proof has more steps than there are protons in the visible universe. But if the logical ought implies can (in the sense of what agents even remotely like us can do), IMP cannot be correct. An analogous objection can be leveled at CON. An agent may find herself in a situation in which she harbors an inconsistent belief set, yet the inconsistency may be so difficult to detect that she cannot reasonably be expected to become aware of it, let alone to take measures for resolving it. We may summarize these objections under the heading *Objection from Excessive Demands*.

(4) There are various types of epistemic situations in which it is arguably permissible or even rationally mandated to have inconsistent beliefs. We can distinguish the following such scenarios.

- As we have just seen, some inconsistencies may simply lie beyond my cognitive reach since detecting them would involve complicated deductions that exceed my skimpy powers of computation. Arguably, in situations in which I am unaware of the inconsistency among my beliefs (and in which I could not reasonably be expected to be aware of it), I cannot be epistemically faulted. In other words the maxim of avoiding inconsistency may also be vulnerable to the Objection from Excessive Demands.
- Suppose that I *do* recognize my belief set to be inconsistent, but that I am unable to restore consistency. I might be unable to resolve the inconsistency because doing so would again require cognitive resources that simply outrun my own, or it might be that I would have to expend more of those resources

on the task in question than it is worth given my goals and interests. I may have more pressing business to attend to than to spend my days trying to weed out a minor inconsistency.

- Finally, I may find myself in epistemic circumstances in which inconsistency is not merely excusable on account of my ‘finitary predicament’ (Cherniak 1986), but where inconsistency appears to be rationally required. Arguably, the well-known Preface Paradox constitutes such a scenario (Makinson 1965).⁷ Here is one standard way of presenting it. Suppose I author a meticulously researched non-fiction book on the subject of cuttlefish. My book is composed of a large set of non-trivial propositions P_1, \dots, P_n about the extraordinary physiology and ethology of cuttlefish. Seeing that all of my claims are the product of scrupulous research, I have every reason firmly to believe each of the P_i individually. But I also simultaneously have overwhelming inductive evidence for Q : That at least one of my beliefs is in error.⁸ The P_i and Q cannot be jointly true; I find myself with a logically inconsistent belief set. Yet, it seems it would seem irrational, in the present situation, to abandon any of my beliefs with a view to regaining consistency. We have already said that the case for believing Q is overwhelming. Surely, though, I cannot simply ditch some of the P_i willy-nilly either.⁹ Indeed, as MacFarlane (op. cit., p. 15) rightly emphasizes, even if, irresponsibly, I wanted to sacrifice some or all of the claims in my book, unless one maintains a wildly far-fetched form of belief voluntarism, it is not something I can simply *decide* to do. Moreover, unless I abandon all but a very small number of the claims in my book, I will again find myself in a similar Preface-like bind. Hence, consistency-restoring belief revision is not an option in the situation described (in the absence of further relevant evidence).

⁷Harman also enlists—more controversially—the Liar Paradox as an example of unavoidable inconsistencies of our beliefs Harman (1986, p. 16). The case of the Liar introduces additional difficulties that would only obscure the issue at hand. I therefore will make no further mention of it in what follows.

⁸It is sometimes said that the fact that Q is ‘second-order’ (a belief about the belief system) somehow undermines the Paradox. I cannot see why this should be so. In any case, (Easwaran and Fitelson 2013) have shown that it is possible to formulate a ‘homogenous’ Preface Paradox that avoids any level distinctions between doxastic states.

⁹Acting in this way would carry the considerable risk of what Niko Kolodny calls satisfying a coherence requirement ‘against reason’ (Kolodny 2007). The idea is this. An agent with an inconsistent belief set can reimpose consistency in a variety of ways that are not sensitive to the evidence she is presented with. Suppose I have good epistemic reasons for believing P and that I also happen to believe $\neg P$ on rather flimsy grounds. As far as the consistency requirement is concerned it does not matter which of the two beliefs I abandon. Yet insofar as epistemic rationality demands that I respect the evidence, it is clear that what, rationally speaking, I should do, is ditch $\neg P$.

The Preface Paradox thus tells against CON; it suggests that I may be within my rational rights in holding inconsistent beliefs (at least in certain contexts). More importantly for my purposes, however, it seems also to constitute a counterexample to IMP. For in the Preface scenario I believe each of the P_i while simultaneously disbelieving an obvious logical consequence thereof in their conjunction (because Q is transparently equivalent to $\neg(P_1 \wedge \dots \wedge P_n)$).

So much for the objections to IMP and CON. It is on the basis of roughly these objections that Harman not only rejects IMP and CON but denies that there are any comparable principles in the vicinity that have any hope of being philosophically viable. His claim appears to be that there simply is nothing interesting and systematic to be said about the normative relation between principles of deductive logic and reasoning.¹⁰ I am interested in whether some version of NT can be defended. In asking this question, my primary focus is IMP; I will set CON aside.

4 Bridge principles

Harman's objections convincingly demonstrate that IMP, in its current formulation, is too crude for the job at hand. The question that will occupy us for the remainder of the paper is whether there is an adequate replacement for IMP that does reveal a normative link between logic and reasoning, but which is invulnerable to the objections of the previous section. In other words, to speak with John MacFarlane (2004), we are asking whether a tenable *bridge principle* is to be had, where by 'bridge principle' we mean a general principle capable of articulating a substantive relation between 'facts about logical consequence' (or our attitudes towards such facts) on the one hand, and norms governing the agent's doxastic attitudes vis-à-vis the propositions standing in these logical relations on the other.¹¹

Now, in order properly to assess Harman's claim that there is no such serviceable bridge principle, we need to know what 'the options are'. That is, to make good on Harman's non-existence claim we need a way of overseeing the range of possible candidate bridge principles so as to ensure that there is no principle we might have missed. But how? Thankfully, a good deal of the work of mapping out the logical space of bridge principles has been undertaken by John MacFarlane (2004), who provides an extremely useful taxonomy of bridge principles. I devote

¹⁰As I will discuss below at the end of section 6, there may be reasons for taking Harman to be making a somewhat weaker claim.

¹¹My talk of logical 'facts' should be taken with a grain of salt. I adopt it mainly for ease of presentation. However, the discussion to follow should be of interests even to non-factualists about logic. The alternatives to IMP we consider can be straightforwardly reformulated in ways that are congenial to the non-factualist (roughly along the lines of Field's 'projectivist' principles (Field 2009a,b, Forthcoming)).

the remainder of this section to describing and further developing MacFarlane’s classification. With its help, we will be able to evaluate Harman’s claim in a systematic fashion.

Let us begin with the general schema—the blue print, if you like—for a bridge principle:¹²

- (★) If $P_1, P_2, \dots, P_n \models Q$, then Φ

A bridge principle takes the form of a material conditional, the antecedent of which states ‘facts’ about logical consequence (or attitudes about such such ‘facts’). The consequent contains a normative claim concerning the agents doxastic attitudes whose contents stand in the said logical relations.

Taking this schema as his point of departure, MacFarlane now introduces three parameters. Each parameter allows for multiple discrete settings. The range of possible combinations among these parameter settings constitutes the logical space of bridge principles. The possible parameter settings are as follows:

1. Bridge principles may differ in the deontic operator they deploy: Does the normative constraint take the form of an *ought* (o),¹³ a permission (p) or merely of having (defeasible) reasons (r)?
2. What is the *polarity* of the normative claim? Is it a *positive* obligation/permission/reason to *believe* a certain proposition given one’s belief in a number of premises (+)? Or rather is it a *negative* obligation/permission/reason *not to disbelieve* (-)?
3. Different bridge principles result from giving the deontic operator different scope. Let O stand generically for one of the above deontic operators. Given that the consequent of a bridge principle will typically itself take the form of a conditional, the operator can take

- narrow scope with respect to the consequent (C) ($P \supset O(Q)$);
- wide scope (W) $O(P \supset Q)$;

¹²We will soon also encounter what I call *attitudinal* bridge principles. They take the slightly different form:

- (★a) If $\alpha(P_1, P_2, \dots, P_n \models Q)$, then Φ

where α designates a specific attitude of a particular agent towards the instance of \models .

¹³The verb ‘ought’ in English has no nominalization (as others have noted, cf. (Broome 2013, p. 8)). I will at times help myself to the noun ‘obligation’, though this is inexact. *Oughts* do not always come with concomitant obligations, at least not on our ordinary understanding of ‘obligation’. For example, it may be that I ought to decorate my office with plants, while not having any obligation to do so. The reader should thus be aware that I am using ‘obligation’ as a term of art so as to function as a nominalization for ‘ought’.

- or it can govern both the antecedent and the consequent of the conditional (B) ($O(P) \supset O(Q)$).

These parameter settings can be combined so as to generate a total of eighteen bridge principles. The symbols in parentheses associated with each parameter setting, combine to determine a unique label for each of the principles: The first letter indicates the scope of the deontic operator (C, W or B), the second letter indicates the type of deontic operator (o(bligation), p(ermisions), r(easons)), and ‘+’ and ‘-’ indicate the polarity.¹⁴ For example, the label ‘Co+’ corresponds to our original principle IMP: ‘If $P_1, P_2, \dots, P_n \models Q$, then if you believe P_1, P_2, \dots, P_n , you *ought* to believe Q ’. ‘Wp+’ designates: ‘If $P_1, P_2, \dots, P_n \models Q$, then it is permitted that (if you believe P_1, P_2, \dots, P_n , you believe Q)’, and so on.

Many will take the bridge principles we have presented thus far to be problematic inasmuch as they are all vulnerable to Harman’s Objection from Excessive Demands because none of the principles is sensitive to the agent’s recognitional capacities. They relate ‘facts’ of logical entailment to certain normative claims regarding the agent’s relevant beliefs. But as we have mentioned, the normative claims are not ones an ordinary agent can satisfy, because the agent cannot be expected to be apprised of all logical ‘facts’.

Consequently, there seems to be a good *prima facie* case for considering bridge principles whose antecedents are restricted to logical implications the agent actually recognizes or believes to obtain. For instance, Co+ now becomes:

- (Co+a) If S recognizes that $P_1, \dots, P_n \models Q$, then if S believes the P_i , S ought to believe Q .

Let us call this the *attitudinally constrained* variant of Co+ (whence the ‘a’ in the label). We may consider different types of attitudes. Recognizing, I take it, is a factive attitude. Internalists about epistemic justification and rationality may be dissatisfied with this. An advocate of mentalism (as opposed to accessibilism) might maintain that epistemic rationality supervenes on the agents *non-factive* attitudes (e.g. (Broome 2012, p. 288) and Wedgwood (2002)). Internalists of that stripe might opt for principles that are relativized to the agent’s beliefs about what follows from what.¹⁵¹⁶

¹⁴See (MacFarlane 2004, p. 7) for an exhaustive list of all bridge principles that can be generated in this way.

¹⁵Externalists typically reject such relativized principles, see for example (Goldman 2009, p. 319). For a general critique of the notion that principles of rationality should always be ‘transparent’ to us, see (Williamson 2000).

¹⁶Notice that belief-like attitudes towards logical consequence might be dispositionally articulated. It is not necessary that the agent explicitly believe every such entailment, it is enough that she should have a disposition to recognize such implications. For a fuller dispositional account of logical beliefs and knowledge and some of problems faced by it, see (Murzi and Steinberger 2013).

Having thus outlined the classificatory scheme, a number of comments are in order. As we have seen, all bridge principles take the form of conditionals. For our purposes all conditionals may be read as material conditionals. Within the present taxonomy, the normative claim in the consequent itself has the form of a conditional. I will refer to it as ‘the embedded conditional’, so as to distinguish it from the ‘main conditional’.¹⁷

MacFarlane’s bridge principles are not fully explicit. Some of the missing information is easily filled in: Bridge principles involve tacit universal quantification over all agents S and all instances of the relation of logical consequence. But there is another underspecified aspect of MacFarlane’s bridge principles, which does merit a clarificatory remark. Consider again Co+a. The principle may in fact be read in two ways. It may be read *diachronically*—that is, as a principle as to how an agent should modify her doxastic attitudes over time in view of the logical relations between their contents; or it may be read *synchronically*—as a principle as to which types of patterns of doxastic attitudes are permissible at a given moment in time. More precisely, the diachronic version of Co+a could be formulated thus:

- (Co+a-Dia) If S recognizes at t that $P_1, \dots, P_n \models Q$, then if S believes the P_i at t , S ought to believe Q at t' (where t ‘immediately’ precedes t').

By contrast, the synchronic variant, when fully spelled out, comes to this:

- (Co+a-Sync) If S recognizes at t or before that $P_1, \dots, P_n \models Q$, then if S believes the P_i at t , S ought to believe Q at t .

Though MacFarlane is not explicit about this point, his discussion makes it clear that the latter synchronic reading of bridge principles is intended. I will follow his lead: In what follows, all bridge principles are to be understood synchronically, allowing us to do away with the cumbersome time parameters.

A further point of clarification regarding the nature of the doxastic attitudes appealed to. *Disbelieving* P is to be distinguished from *not believing* P . One cannot rationally believe and disbelieve the same proposition.¹⁸ Hence, I ought

¹⁷Notice that other types of bridge principles are conceivable. For instance, one might seek to extend MacFarlane’s taxonomy by considering also primitive binary conditional deontic operators, i.e. operators that are not decomposable into a material conditional and a unary deontic operator. I have investigated operators of this kind in the context of logical bridge principles in (Steinberger Forthcoming, §5). I will not do so here for some of the reasons I discuss in that paper. See also (Kolodny and MacFarlane 2010, p. 126) for related worries about such operators.

¹⁸I am setting aside Fregean puzzles here: On a more course-grained conception of propositions the propositions expressed by ‘Superman can fly’ and ‘Clark Kent can fly’ might be taken to be one and the same. Yet, Louis Lane does not appear to be irrational if she believes that which is expressed by the former sentence, but disbelieves what is expressed by the latter sentence.

to ensure that when I disbelieve P , I do not believe P . The converse, however, obviously does not hold since I can fail to believe P without actively disbelieving it. I may, for instance, choose to suspend my judgment as to whether P pending further evidence, or I may simply never have considered whether P .

Let us turn now to the deontic modals occurring in our bridge principles. ‘You ought not Φ ’ ($O\neg\Phi$) is not the same as saying ‘It is not the case that you ought to Φ ’ ($\neg O\Phi$). But rather ‘You are forbidden from Φ ing’. Consequently, ‘You ought not disbelieve P ’ should be read as ‘You are forbidden from disbelieving P ’, as opposed to ‘It is not the case that you ought to disbelieve P ’, which is compatible with it being permissible to disbelieve P .

Moreover, *ought* and *may* are understood to be strict or ‘all-things-considered’ notions, *reason* is a *pro tanto* or contributory notion. Having a reason to ϕ is compatible with simultaneously having reasons not to ϕ and even with it being the case that I ought not to ϕ . Reasons, unlike *oughts*, may be weighed against each other; the side that wins out determines what ought to be done. Hence, it may be perfectly proper for me to have reasons to ϕ and yet not to ϕ because my reasons are overridden, whereas if I ought to ϕ and fail to do so, this amounts to a normative failure on my part. I shall not assume that conflicting ought-claims are impossible as some of the great philosophers have done in the past (at least in the moral domain), nevertheless clashing obligations differ fundamentally from competing reasons in that any agent in such a situation is inevitably liable to criticism.

Finally, I will treat all deontic modals as propositional operators. This too is not uncontroversial. Peter Geach (1982) and more recently Mark Schroeder (2011) have argued that so-called deliberative or practical *oughts* are best analyzed not as operators acting on propositions but rather as expressing relations between agents and actions.¹⁹ Nevertheless I will assume without argument that the operator-reading can be made to work (for defenses of this position see e.g. (Broome 2000, 2013) and (Wedgwood 2006)). We can capture the particular connection between an agent and the obligation she has towards a proposition at a particular time, by indexing the operator: $O_{S,t}$. I will drop the indices in what follows.

The framework provides a wide array of candidate bridge principles. But how are we to assess our candidates? MacFarlane proposes the following procedure. He lays down a number of criteria which, in his estimation, a good bridge principle should meet. He then evaluates each of the contenders against these desiderata. The winner (or set of winning principles) is to be determined by way of a cost-benefit analysis on the basis of each principle’s performance as judged against the criteria.

It is here that the objections we encountered in section 3, make their second

¹⁹MacFarlane (2014, Ch. 11) has followed suit.

appearance; this time in the role of criteria of adequacy. But the challenges posed by these objections are not the sole desiderata MacFarlane considers. He also proposes the following two additional desiderata:

- **The Strictness Test:** At least when it comes to ordinary, readily recognizable logical implications leading to conclusions that the agent has reason to consider, there is something amiss about an agent who endorses the premises but fails to believe the conclusion (MacFarlane 2004, p. 12). (MacFarlane takes inspiration from (Broome 2000, p. 85).)
- **The Priority Question:** The attitudinal variants have a distinctive advantage when it comes to dealing with Excessive Demands worries. But relativizing one's logical obligations to one's logical beliefs or one's logical knowledge invites problems of its own, according to MacFarlane.²⁰ The problem according to MacFarlane is that 'we seek logical knowledge so that we will know how we ought to revise our beliefs: Not just how we will be obligated to revise them when we acquire this logical knowledge, but how we are obligated to revise them even now, in our state of ignorance' (MacFarlane 2004, p. 12).²¹

These are the criteria. MacFarlane's own evaluation in light of these criteria leads him to adopt a combination of the principles $Wr+$ and $Wo-$. I do not wish to engage directly with MacFarlane's conclusions or with the reasoning by which he arrives at them at this point. Instead, I want to draw attention to a three-fold distinction between different roles norms in general, and logical norms in particular, might play. It is only once this distinction is taken on board, I argue, that we can reasonably hope to undertake a meaningful cost-benefit analysis of the sort MacFarlane is after. More generally, the insight that there are *several* broadly normative roles logic might be thought to play in reasoning is of central importance. By distinguishing these three roles we arrive at a fuller, more nuanced understanding of NT, thus shedding new light on the debate. Positively, we will be in a position to formulate and to answer questions of the form: Is there a philosophically defensible bridge principle that vindicates NT understood in this

²⁰MacFarlane (2004) only considers factive attitudes towards logical entailment.

²¹In fact, MacFarlane (p. 12) the following further criterion:

- **Logical Obtuseness:** Suppose someone professes to believe P and Q , but refuses to take a stand (neither believes nor disbelieves) the conjunction $P \wedge Q$. Intuitively, such a person is liable to criticism. However, according to our bridge principles with negative polarity, she is not to be faulted. This may indicate that bridge principles of negative polarity are ultimately too weak.

For my purposes Logical Obtuseness, which tells mainly against bridge principles of negative polarity, is of negligible importance, which is why I put it to one side here.

particular way? The following section introduces the distinction between three types of norms and applies them to the bridge principles.

5 Three normative roles

Norms, quite generally, can serve different purposes. Of particular interest for our present concerns are the following three distinct roles norms can play. Norms may serve as

1. *directives* which may guide a subject in deliberating over what to do, choose or believe.
2. *evaluations* by setting standards by which to assess acts, states or the like as, in some sense, good or bad, correct or incorrect.
3. *appraisals* by laying the basis of our attributions of praise or blame to others (or ourselves).

Of course, these normative roles are not wholly independent of one another. Typically, we follow normative directives with an aim to conduct ourselves in a way that meets or at least approximates an evaluative standard. And we appraise one another with a view how well, given the standard set by our evaluative ideal, we reason given our epistemic situation.

The main claim of this paper, we said, is that the question of the status of NT—that is, the question ‘In what sense is logic normative for reasoning?’—is ambiguous between these three senses of ‘normative’. We are now in a position to disambiguate the question. That is, we are now in a position to appreciate that there are in fact (at least) three distinct questions concerning the normative status of logic one might ask: One corresponding to each of the three normative roles in play. The ambiguity equally carries over the question we have been asking. The question, namely, ‘Is there a philosophically viable bridge principle?’ Let us, then, elaborate on our three-way distinction between normative roles, applying them to our bridge principles.

1. The first sense in which logic might be thought to be normative is, as we have already seen, that of offering a directive prescribing what, epistemically speaking, the agent ought to ‘do’. So understood, the function of a bridge principle is to articulate the precise ways in which logic should inform the agent’s doxastic deliberations. It does so by issuing instructions *guiding* the agent in forming and revising her beliefs in ways that appropriately respect logical strictures. Call a bridge principle of this type a *directive* or *guiding bridge principle*. It is characteristic of directives that they should operate at the level of first-person deliberation.

According to such principles, it is at this first-person level that the normative impact of logic makes itself felt: Given my current epistemic situation as it presents itself to me, how should I be managing my beliefs in light of the logical relations between them (i.e. their contents)? Clearly, a bridge principle can fulfill this guiding role only if the agent is actually able to apply it. In order for an agent to be able to actually to apply a rule, it must be transparent to her, when the conditions for its application obtain. In particular, in order to be able actually to apply a bridge principle, she must be in a position to recognize whether the logical relations mentioned in the antecedent of the principle obtain or not. It is plausible, therefore, that guiding principles must be such that the logical relations adverted to in their antecedent, must be relativized to the agent's non-factive attitudes, typically her beliefs concerning logical entailment. Principles that are to serve this directing function thus will take the form of attitudinal bridge principles:

If S believes that $P_1, \dots, P_n \models Q$, then...

However, they will only have this form when regarded 'from the outside' so to speak. From the agent's own perspective, questions like 'Do I believe that Q follows from P_1, \dots, P_n ?' are transparent to the question 'Does Q follow from P_1, \dots, P_n ?' In following the norm, I wonder what follows from what, not what I take to follow from what.

2. Alternatively, according to the second interpretation of NT, we might be after evaluative bridge principles, or *evaluations* for short. Principles of this sort function, as their name makes plain, as evaluative yardsticks. They set the standards against which to assess the doxastic states of others according to the logical properties and relations holding of and between the contents of these states. Unlike directives, evaluations operate from a third-person standpoint. (Though we may of course evaluate our own doxastic states.) Since their aim is neither that of guiding the agent, nor that of attributing praise or blame, evaluative principles are not sensitive to what the agent under evaluation believes to follow from what. In other words, the principles need not be ones that the agent is always in a position to follow. All that matters from the vantage point of an evaluative principle is whether, objectively speaking, the agent's doxastic state does or does not conform to the strictures of logic in the ways required by the bridge principle. To emphasize the point, norms of this type are objective in that they classify states into logically correct and incorrect ones by the standards of the demands of logic, not by the standards of what the agent takes the demands of logic to be. Of course it may be that any actual application of an evaluative principle will again be relativized, but it will be relativized this time not to the agent being evaluated, but to the evaluator. In evaluating someone we can only go on what we take the 'logical facts' to be, not by logical facts as they are by themselves, in part unbeknownst to us (if there is any such thing). However, the evaluator takes her own judgments as to

what follows from what to be answerable to the ‘logical facts’. That is, according to the bridge principle what ultimately determines which doxastic states are admissible and which are defective are the entailment relations whether or not the agent being evaluated or the evaluator or indeed anybody else is aware of these entailment relations.

3. There is a third type of normative role we have already touched upon—the kind that deals in praise and blame. Call them appraising bridge principles, or *appraisals* for short. Like evaluative bridge principles, this type of appraisal is made from a third-person point of view (though they may also take the form of second person advice). However, unlike evaluations, appraisals are concerned with agents can be held responsible for. Agents can be held responsible only for that of which they can be aware, appraisals of this sort amount to ‘by-the-lights-of the agent’-evaluation. That is to say, appraising principles, like guiding principles, are sensitive to how things present themselves from the viewpoint of the agent. The agent can only be held responsible for failing properly to take into account those logical entailments that she is aware of (or at least those that she can be expected to be aware of given her logical beliefs). As the last parenthetical remark indicates, appraising principles differ from directives not only by taking a third-person standpoint, but also in that they may introduce varying degrees of idealization, depending on how much logical acumen is deemed exigible from our agent. At one extreme—degree zero of idealization, if you like—an appraising principle just is relativized to the agent’s actual logical beliefs. The agent’s logical obligations extend no further than her momentary state of logical competence. At the other extreme—maximal idealization—the agent is expected to be disposed to recognize any logical entailment (or at least all the logical entailments believed by the assessor). At degree zero, appraising principles mimic, from a third-person point of view, the corresponding directives; at the maximal level of idealization, appraising principles converge with the corresponding evaluative principles (one is, at it were, appraising a potentially logically perfect agent). Typically, appraising principles operate at an intermediary degree of idealization. For instance, even when we are assessing an agent by her own ‘logical lights’, we may abstract away from whimsical logical beliefs that the agent would readily revise or recognize were she to pay the matter sufficient thought. How much idealization is permissible? For instance, might we even correct for systematic errors in logical reasoning like those frequently documented in the findings of cognitive psychologists? Presumably there are no firm rules here. Everything depends on what degree of idealization the evaluator deems most useful in the context in question.

An example of Field’s will help illustrate these distinctions. ‘Suppose I have after laborious effort proved a certain unobvious mathematical claim, by a proof formalizable in standard set theory, but that you don’t know of the proof’ (Field

Forthcoming, p. 12). Assuming, as we will, that you firmly believe the standard axioms of set theory (ZFC say), there is a clear sense in which you ought to believe the claim in question. This is the evaluative or strictly objective sense of ‘ought’. However, there is another equally intuitive sense in which it is not the case that you ought to believe the claim. After all, the claim and its proof is entirely unobvious; you cannot possibly be expected to recognize the claim as following from the axioms and so cannot be faulted for failing to see the entailment. This is the subjective sense of ‘ought’ we encounter in appraising principles: If you are unaware of any reasons for believing the claim, it seems wrong to say that you ought to believe it. ‘Ought’, here, is understood so that (very roughly) believing the claim is the attitude you have most reasons to adopt in light of what you take to be the case (i.e. what you take to be entailed by the propositions you believe).²² Finally, also if our bridge principle is playing a guiding role, endorsing the unobvious theorem is certainly not what you ought to do. The ‘fact’ that, unbeknownst to you, the claim in question is a logical consequence of axioms you endorse, is irrelevant to your deliberation; the only facts that can enter into your deliberation are ones of which you are apprised. Hence, a guiding bridge principle can never give you reasons for believing the claim (though of course it is possible that you have extra-logical reasons for believing it; the testimony of others, say).

So much for the application of our distinction to bridge principles. The distinction between different normative roles of bridge principles is, as I noted at the end of the previous section, crucial when it comes to assessing bridge principles. It seems clear that our distinction must factor into MacFarlane’s cost-benefit analysis. All the desiderata MacFarlane considers are categorical. They are either met or fail to be met. They cannot be both met and not met, nor can a desideratum be met to a greater or lesser extent. All the same, the desiderata might plausibly be given more or less weight. But according to what criteria do we determine their importance? It is at this stage that the normative function the bridge principle is to serve—whether it is advanced as a directive, an evaluation or an appraisal—is of central importance. For instance, the Objection from Excessive Demands appears to rule out certain bridge principle on account of placing unreasonable demands

²²In fact, not only might it be excusable for you not to believe the claim in question, it might be thought to be positively epistemically irresponsible of you to believe the claim if you do not believe it *for* the right reasons. The fact that you believe the axioms and that the axioms entail the claim does provide you with good reasons in a certain objective sense. But, *ex hypothesi*, you are not believing the claim *for* those reasons (or any other good reasons). Therefore, from this perspective, it is not merely not the case that you ought to believe the claim, you positively ought not believe it. Your belief in the claim is, in the vocabulary of Feldman and Conee, is not *well founded* (Feldman and Conee 1985, p. 24). Another familiar way to make the same point is to say that the claim is *propositionally* but not *doxastically* justified for you. That being said, all that matters for present purposes is that there is a clear sense according to which it is not the case that you ought to believe the claim.

on our ability to recognize logical implications. The objection may be entirely appropriate provided the bridge principle is to play the role of a directive or an appraisal. However, it would seem to be beside the point if the principle is to underwrite a conception whereby logic plays an evaluative role. Bridge principles will thus be more or less strongly responsible to different desiderata depending on the kind of normative role one wishes them to have. Consequently, a given principle may—on account of its structure, i.e. its particular parameter setting—be a better fit for one type of normative role than another. All this should be reflected in our cost-benefit analysis.

One aspect of the picture we just painted is inaccurate. Given what I just said, it may seem as if one and the same principle can be assessed *qua* directive, *qua* evaluation, and so on. But this is not quite right. A given principle must be interpreted in rather different ways, depending on the type of assessment we have in mind. Thus, when we say that a principle like, $Wr+$ say, is better suited to play the role of directive than it is to play that of an evaluation, we are not strictly speaking assessing one and the same principle against differently weighted desiderata. Rather we are comparing two slightly different principles, namely $Wr+$ interpreted as a directive and $Wr+$ interpreted as an evaluation. The main difference between these principles resides in the reading of the deontic modal. Each normative role goes hand-in-hand with a corresponding interpretation of the deontic modals occurring within the bridge principle so interpreted. (For simplicity, I will focus on *ought*, but everything I say carries over *mutatis mutandis* to the other deontic modals.) The *ought* figuring in a bridge principle interpreted as a guiding principle is naturally read as what is generally referred to in the literature as the *practical* or the *deliberative ought*.²³ Roughly, I take *oughts* of this sort to allocate responsibility for an action to an agent (as in e.g. ‘Noa really ought to call her mother’). Among its characteristic features is that it is of the *can-implying* variety and that it is immediately relevant to deliberation. In contrast to such an ‘ought-to-do’, in Sellars’ terminology, the *ought* we encounter in evaluative principles (i.e. bridge principles interpreted as evaluations) is best thought of as an ‘ought-to-be’ or, appropriately, an *evaluative ought*. Evaluative *oughts* present a certain state of affairs as desirable without necessarily holding anybody responsible for bringing the said state of affairs about, as in: ‘There ought to be playgrounds in urban areas’. Unlike deliberative *oughts*, an evaluative *ought* is neither *can-implying* or directly relevant to our deliberations.²⁴

Finally, the *oughts* in appraising bridge principles are ones, which, depending on the degree of idealization in question, are more or less subjective, that is,

²³See (Schroeder 2011, §2.1) for a fuller characterization of deliberative *oughts*. See also Broome’s discussion of what he calls the ‘central ought’ (Broome 2013, §2.3).

²⁴See (Sellars 1969) and (Humberstone 1971) for the underlying distinction.

relativized to a set of beliefs that is more or less keyed to the agent's actual set of beliefs, as opposed to the set of beliefs the agent would have under more favorable circumstances. *Oughts* thus seem to come in more or less 'subjective' or more 'objective' flavors. There has been much debate in recent literature about how best to account for the semantics of deontic modals so as to adequately capture this feature. I do not wish to take sides on the issue here; everything I say should be compatible with all of the accounts on offer. All that matters here is that there is *some* semantic story to be told that succeeds in tracking the sensitivity of deontic modals to the agent's informational state and possibly the informational states of other agents advising or evaluating her (as in MacFarlane's treatment of deontic modals as assessment sensitive expressions (MacFarlane 2014)). The

With these distinctions in place, let us now look at some of the different ways of understanding NT. To begin, let us put the distinctions we have just introduced to good use, in taking a second look at Harman's case against NT. The question that occupies Harman (and to which he appears to offer a negative answer) is whether there is an interesting systematic normative connection between principles of deductive logic and a 'theory of reasoning'. As we have noted in section 2, by a 'theory of reasoning' he means a normative account of how, from the first-person perspective, the agent should go about managing—forming, revising, or perhaps reinstating—her beliefs in light of her interactions with the world and her resulting epistemic situation. It is clear, therefore, that Harman interprets NT as the thesis that there is an acceptable *guiding* principle to be had. His claim, I take it, is that there is no such principle.²⁵

Let us contrast this with two defenses of NT, those of MacFarlane (2004) and of Field (2009a, Forthcoming). What our threefold distinction brings to light, is that MacFarlane's and Field's proposals, for all their considerable merits, could not possibly provide solutions to the problem *as Harman conceives of it*. Let us consider them in turn.

It is not entirely clear which of the three types of normative roles he seeks to capture, or indeed whether his position is stable. He appears to waver between the two non-guiding types of normative assessments. His insistence on the question of the agent's responsibility—as manifested by the weight he gives to the Excessive Demands worry—suggests that he conceives of bridge principles as performing an appraising role. At the same time, he does not consider attitudinal principles aside from factive ones, which he rejects. Moreover, the fact that he deems the Priority Question to tell decisively against his k-principles, shows that he believes logical entailments to have normative force even when the agent does not recognize or

²⁵He is perfectly explicit that any candidate bridge principle would have to have the hall marks of a directive. In particular, it would have to comprise 'subjective' deontic vocabulary and be attitudinally restricted (Harman 2009, p. 333). See also Harman's summary of his own epistemological views (Harman 2010).

believe them to obtain. This suggests that he is thinking of bridge principles as fulfilling an evaluative function.

Field, on the other hand, is clear that the bridge principle he proposes is to be understood as fulfilling an evaluative role. At least, he is explicit about this in the latest version of his account, where he notes ‘that we recognize multiple constraints on belief, which operate on different levels and may be impossible to simultaneously satisfy’ (p. 13). His appeal to the ‘non-subjective sense of “should”’ makes it plain that he is after what I call the evaluative role. My tripartite distinction is an attempt to pin down and characterize more fully the ‘multiple constraints’ operating ‘at different levels’ mentioned by Field. It is noteworthy, however, that in earlier incarnations of his account (Field 2009a,b), Field seems less clear about this. He is there clearly troubled by the Excessive Demands Objection and flirts with various (factive) attitudinal principles to cope with it.

Thus, when viewed against the backdrop of these distinctions, it becomes apparent that Harman and his opponents are not addressing the same question; NT is understood in subtly but importantly different ways. This does not, *per se* tell against any of the three accounts, of course. What it does show is that accounts like those of Harman and Field that appear to be directly opposed to one another, in fact are not in competition at all. It is conceivable that Harman is right in that there simply is no bridge principle capable of offering the right kind of normative guidance. That claim, however, is perfectly compatible with an affirmative response to the question of logic’s normativity when the latter is understood, for instance, in the following terms: Does logic have an evaluative role to play? A proposal like Field’s might provide such a response. We would then have to conclude that Harman’s skepticism is partially justified in the sense that there is no guiding bridge principle that fits the bill and hence that logical consequence does not directly enter into first-person doxastic deliberation in ways we might have expected. Yet, it would still be true that logic *does* have a broadly normative role to play—to wit, an *evaluative* one.

Now that we have these distinctions in place, we can ask the question whether there are bridge principles, which, when appropriately interpreted, successfully play the role of a directive/evaluation/appraisal. I first turn to the question whether a bridge principle is to be had that is consonant with Harman’s aims and background assumptions. I argue there is a viable and informative guiding bridge principle, albeit a rather weak one.

6 A Harman-friendly principle

We have just seen that Harman interprets NT as a thesis committing us to the existence of a defensible logical directive. Before we embark upon the quest for a

philosophically viable that fits this description, we must briefly turn to a further commitment of Harman’s that will play a role in the remaining discussion. Harman in fact imposes a further constraint on the range of admissible bridge principles. His additional demand is that bridge principles be *qualitative*. More precisely, Harman requires that the doxastic attitudes appealed to in a bridge principle be ‘full beliefs’. This is relevant in particular against the background of the puzzle posed by epistemic paradoxes like the Preface. A standard response to the Preface Paradox consists in appealing to graded credal states in lieu of ‘full’ (‘qualitative’, ‘binary’, ‘all-or-nothing’, ‘outright’) beliefs. ‘Credences’ or ‘degrees of belief’ are typically modeled by means of a (possibly partial) credence function (which we will denote by ‘*cr*’) that maps the set of propositions into the unit interval. *Probabilists* further maintain that an ideally rational agent’s credence function ought to be (or at least ought to be extendable to) a probability function (i.e. it ought to satisfy the standard axioms of probability theory). In other words, an ideally rational agent should have probabilistically coherent credences.²⁶ Probabilists have no trouble accounting for the Preface phenomena: The subjective probability of a large conjunction may well be low—even zero, as in the case of the Lottery Paradox—even if the probability assigned to each of the individual conjuncts is very high (reflecting the high degree of confidence the author, in Preface case, rightly has in each of her claims). A tempting strategy for making sense of NT in light of the Preface Paradox, might therefore consist in following this general approach of appealing to degrees of belief. However, rather than going in for probabilism (directly or at all), the idea would be to devise bridge principles according to which logical principles directly constrain the agent’s degrees of belief (as opposed to constraining her full beliefs). Indeed, Field’s favored bridge principle, which I discuss below, is of this variety.

My point here, however, is that for Harman the answer cannot lie with a quantitative principle. The reason is that he rejects the notion of degree of belief (at least when degrees of belief are understood as graded doxastic states as just described) altogether and quantitative principles along with it. According to him, ‘people do not normally associate with their beliefs degrees of confidence of a sort they can use in reasoning’ because ‘it is too complicated for them to do so.’ Thus, he is led to conclude that ‘the principles of reasoning are principles for modifying [...] all-or-nothing representations’. But what, then, are we to make of the phenomenological datum that beliefs can be held with varying levels of confidence? Harman admits that our beliefs come in ‘varying strengths’, but he denies that it is necessary to postulate the existence of a corresponding graded doxastic state.

²⁶To fix terminology: Bayesianism, as I understand it, is probabilism plus the diachronic rule of Bayesian conditionalization (or some variant thereof) for updating one’s credences in the light of new evidence.

Rather the phenomenon can be ‘explained as a kind of epiphenomenon resulting from the operation of rules of revision’ [e.g. you believe P to a higher degree than Q iff it is harder to stop believing P than to stop believing Q]’ (Harman 1986, p. 22). These rules of revision need only appeal to ‘beliefs one accepts in a yes/no fashion (idem).’ In certain cases, the phenomenon may also be accounted for by attributing full beliefs with explicitly probabilistic contents to the agent.

Harman’s eliminativism about degrees of belief is not, I take it, widely shared. However, the problem posed by Harman is of interest even to those who are more favorably disposed towards the notion of degree of belief. For even an advocate of credence-talk may still be interested to know whether a viable *qualitative* guiding principle can be found. There are at least two motivations. The first calls for some explanation. Some probabilists have argued that epistemic paradoxes like the Preface reveal the concept of full belief with its attendant system of logical norms of rationality to be incoherent (or at the very least unfit to play a role in a rigorous account of rationality). According to this line of thought we should either abandon the concept of full belief altogether or at least be keenly aware of its highly approximate, folk-theoretic character. Thus even if we cannot or will not dispense with full belief talk in every day contexts, it is the concept of degree of belief and the broadly Bayesian machinery underpinning it that alone ultimately enjoys philosophical and scientific respectability.²⁷ In light of the dialectic just sketched, those philosophers seeking to rehabilitate the concept of full belief will have a keen interest in the question as to whether there are qualitative bridge principles capable of dealing with the epistemic paradoxes. The second motivation is tied to our customary mode of representing credences as point-valued functions. The thought is that quantitative principles that require the agent to keep track of and compute precise numerical values simply are not ones agents equipped with computational abilities anything like ours could possibly be guided by. Thus, credence-based norms could not possibly play the role of a guiding principle.

Back to Harman. What is needed to vindicate NT on Harman’s reading and given his background assumptions, is a bridge principle featuring full beliefs, offering genuine directives to reasoners and which meets our desiderata. What form might such a principle take? Interestingly, Harman himself points us in the right direction. In his (Harman 1986, p. 17) he considers the following principle (roughly corresponding to Cr+k in MacFarlane’s classification).

- (Cr+a) If S believes that $P_1, \dots, P_n \models Q$, then S believes all of the P_i only if S has reasons to believe Q .

²⁷Richard Jeffrey (1970, p. 132) famously takes what appears to be a radical eliminativist stance. Christensen (2004) takes a more moderate line with respect to full belief, taking it to be a useful and perhaps indispensable way of talking in many ordinary contexts, but still not one that ‘will turn out to be an important part of epistemic rationality’ (p. 100).

First off, notice that the antecedent of the principle is restricted to relations of logical entailment that the agent believes to obtain. This comes as no surprise since, as we have seen, directives must be attitudinal at least when stated from the ‘outside perspective’. Next, notice that the principle features the weaker *reasons* operator. The defeasible nature of the *reasons* operator is the key to Cr+a’s (relative) success. Consider, for instance, the Too Much Instruction Objection: My having a defeasible reasons for believing Q as a result of Q ’s being entailed by my antecedent beliefs is perfectly compatible with my simultaneously having good, perhaps better, reasons for not believing Q (e.g. the available evidence supports not- Q). This has some initial plausibility: My recognizing that a certain proposition Q is entailed by my beliefs gives me (defeasible) reasons to believe Q , but these reasons can be trumped by weightier epistemic reasons for disbelieving Q .

But what about Broome’s related objection? Broome’s Objection in this context amounts to this: I have reasons for believing any of the beliefs I in fact believe. Is this a tolerable consequence? Many philosophers, I suppose, would deny that it is. To address this worry, I suggest we move to the corresponding wide scope principle:

- (Wr+a) If S believes that $P_1, \dots, P_n \models Q$, then S has reasons to (believe all of the P_i only if S believes Q).

Applied to Broome’s Objection, the wide scope principle yields that, for any proposition, P that I happen to believe, I have a reason to (believe P only if I believe P). This is equivalent to my having reasons to (believe P or not to believe P). Provided one accepts, as I do, that deontic modals are best construed as propositional operators, this causes no problem: The proposition in the scope of the *reasons* operator is a tautology. Hence, nothing at all is required of the agent in order to appropriately take the reasons for believing the embedded tautology into account. A reason to ‘see to it’ that a certain tautology obtains is vacuous—there simply is nothing we need to ‘do’.²⁸ In this way Broome’s Objection can be avoided.²⁹

²⁸Notice, that the principle does not enjoin me to take a stance with respect to P —it does not say that I have reasons to either believe or disbelieve that P , but merely that I have reasons to either believe it or not to believe it (which includes the option of either suspending judgment or simply not considering P at all). Notice also that the formulation ‘having reason to see to it that’ above is simply owed to convenience of exposition. I am not thereby endorsing a ‘*stit*’ reading of deontic modals.

²⁹Interestingly, the Cr+a-induced consequence in the Broome case—that one has reasons to believe any proposition one in fact believes—arguably is acceptable against the background of Harman’s own epistemological views. What I have in mind here is what Harman has dubbed a ‘general foundations theory’ in epistemology (Harman 1986, 2001, 2010). General foundations theories are characterized by the fact that they promote a kind of innocent-until-proven-guilty

We are thus left with Wr+a. How does our principle fare against the remaining objections? Let us start with Excessive Demands. Seeing that we have set ourselves the goal of formulating a guiding principle, the Objection from Excessive Demands is a serious concern. (Not so in the case of evaluative bridge principles as we will see.) However, the objection is neutralized by restricting the principles to logical consequences the agent believes to obtain.

What about about the Objection from Clutter Avoidance? There may be a great many logical consequences I am disposed to recognize, but which I have good reason not to come to explicitly believe because they are of no interest to me and coming to believe them would just needlessly ‘clog up my belief box’. With just a little tweaking Wr+a can tackle the problem:

- (Wr+a*) If S believes that $P_1, \dots, P_n \models Q$ and S has reasons to consider or considers Q , then S believes all of the P_i only if S has reasons to believe Q .

Notice that it would not be enough merely to add the clause ‘and S considers Q ’ because there may be cases in which an agent fails to consider logical consequences she has good reasons to consider. For example, I might, out of intellectual dishonesty, fail to take into account a damning consequence of my philosophical position of which I am otherwise aware. Surely, in such a case, I have good epistemic reasons (in the subjective sense) to consider it.

This should take care of Clutter Avoidance worries. Let us now turn to the Preface Paradox, which spells so much trouble for stronger qualitative bridge principles. As it turns out, it too can be overcome thanks to the weaker defeasible *reasons* operator: I believe each of the propositions composing my book. This provides me with a reason to believe in the conjunction of those propositions, which is a trivial logical consequence and which I both recognize to obtain and have reasons to consider. However, my reason for believing the conjunction can be trumped by my weightier epistemic reasons for believing in my own fallibility. In particular, it can be trumped by my overwhelmingly strong grounds for believing that I have made a mistake somewhere along the way. Thus, the *reasons* operator allows our qualitative principle to accommodate exceptional Preface-like situations in which

policy in matters of belief maintenance. The idea behind such approaches is a form of conservatism about belief: An agent’s belief set enjoys a kind of default justification until she encounters sufficiently strong countervailing evidence. It stands opposed to ‘special foundations theories’ which are conservative only with respect to a privileged proper subset of the overall belief set. On a view like Harman’s, it seems proper to say that I have reasons to believe any proposition I in fact believe. I have reasons to stick to my beliefs unless and until I am presented with sufficiently strong grounds for abandoning them. Consequently, Cr+a is arguably immune to Broome’s Objection if we accept Harman’s own general foundations theory. However, I believe there are good independent reasons for preferring Wr+a over Cr+a.

our standard logic-induced epistemic obligations are suspended (because they are overridden by competing epistemic considerations).

Let us briefly take stock. Our review of the objections and challenges faced by a bridge principle of Harman’s specifications, led us to the catchily named principle $Wr+a^*$ —a principle that has a lot to recommend itself. Before embracing it wholeheartedly, however, we need also to consider MacFarlane’s additional desiderata: The Strictness Test and the Priority Question. I discuss them in reverse order.

The Priority Question was supposed to tell against attitudinally restricted bridge principles. The idea was that the reason we seek to increase our logical knowledge is precisely because in doing so we are finding out about the ways in which we are normatively bound by logic even now, in our state of relative logical ignorance. The trouble with this objection is that it stems from a failure to distinguish between guiding and evaluative bridge principles. To be sure, there is a strong intuition that we are answerable to the laws of logic even when we are ignorant of those laws. However, this intuition is associated with evaluative (and perhaps appraising) bridge principles. From the deliberative first-personal standpoint of guiding bridge principles, logical facts that lie beyond the momentary cognitive reach of the agent are a non-factor. Consequently, MacFarlane’s objection to attitudinally restricted principles evaporates once the different modes of normative evaluation are clearly distinguished. All this is perfectly compatible, of course, with the meta-principle that, as a rational agent, I should continuously strive for greater logical competence.

Let us turn to the final hurdle: The Strictness Test. The thought, recall, is that at least when it comes to obvious logical entailments leading to conclusions that the agent considers or has reasons to consider, an agent who endorses the premises but fails to believe the conclusion opens himself up to criticism. Principles based on the *reasons* operator fail the Strictness Test. The flexibility allowing them to parry Preface Paradox-like challenges—namely, the fact that they allow for cases in which our reasons for logical coherence are trumped by other epistemic considerations—now turns out to be the source of weakness. But the blemish is not to be avoided. So long as one is wedded, as Harman is, to the idea that bridge principles may only involve full beliefs, it looks as if the only way around the Preface Paradox consists in resorting to a weaker, non-strict deontic operator.

The upshot is that as far as qualitative guiding bridge principles go, we are limited to the following options: We must either try to parlay ourselves out of our ‘Preface intuitions’, or we must learn to live without Strictness. There are, of course, a number of clever philosophers who believe the Preface Paradox admits of a resolution purely in terms of all-or-nothing beliefs. There is a strict qualitative guiding bridge principle available to such philosophers:

- ($Wo+a^*$) If S believes that $A_1, \dots, A_n \models C$ and S has reasons to consider C

or considers C , then S *ought* to (believe all of the A_i only if S believes C).

It can easily be checked that the principle performs equally well as its defeasible counterpart with respect to all of the desiderata we have considered with the sole exception that it succeeds where $Wr+a^*$ failed (and vice versa); that is, the strict $Wo+a^*$ fails to respect our intuitions in the Preface Paradox case while passing the Strictness Test. I wish I could join the Preface Paradox optimists. Thus far, however, I have been unable to convince myself to do so.³⁰

An alternative response, MacFarlane's (2004, p. 14) response, is to accept the Preface intuitions but to resist the idea that they show our strict bridge principles to be too strong. According to MacFarlane, we simply must reconcile ourselves to the existence of an ineliminable normative conflict: Our strict logical obligations clash with other epistemic obligations, namely, the obligation to believe that some of my beliefs must be mistaken. Our agent becomes a tragic heroine. She finds herself, through no fault of her own, in a situation in which, no matter what she does, she will fall short of what, epistemically speaking, she ought to do.

However, as a matter of sound methodology, admitting an irresolvable normative clash should only be our last resort. A better approach, it seems to me, would consist in finding a way of reconciling the conflicting epistemic norms. But how? Here is a simple idea. Let us assume we have a partial ordering over our norms according to priority. We could allow the priority ordering to vary with the contexts. For instance, though norm N_1 may take precedence over N_2 in one context, but N_2 may have a higher priority than N_1 in another. For instance the norm of being polite may be judged to be more important than that of being honest in some situations (when it comes to your opinions about friend's baby's cuteness say); all the same, there are important situations in which we regard honesty as vastly more important than politeness.

How does this thought apply in the case of the epistemic norms that interest us? Our intuitions regarding strictness apply in *ordinary* contexts, where the agent is typically dealing with a very limited number of beliefs and their manageable logical consequences. In situations like these our Preface intuitions get no purchase on us. Indeed, it is highly plausible that (in Preface-like contexts where we are considering claims that are the outcome of scrupulous research and enjoy a correspondingly high degree of confidence) the fewer the propositions we are dealing with, the less plausible it is that any of them should be false. Our Preface intuitions only kick in once we deal with sufficiently large bodies of propositions. Conversely, it is precisely in the contexts in which our Preface intuitions get a foothold, that our intuitions regarding strictness lose their force. In such Preface Paradox-like scenarios, it is rationally required, or at least excusable, to violate strictness.

³⁰See (Christensen 2004, Ch. 3) for an illuminating discussion of the significance and the inevitability of the Preface Paradox.

In light of this, what we would like is a principle that is sensitive to whether the agent's situation is 'ordinary' or 'Preface-like'. More precisely, we would like instructions as to how our logic-induced *reasons* (as expressed by $Wr+a^*$) are to be understood. How heavily should one weight the reasons for respecting the demands for logical coherence, as opposed to the reasons favoring competing epistemic considerations *depending on* whether the context is ordinary or Preface-like? In ordinary contexts our reasons for taking into account the logical consequences (the ones we believe to be consequences and that we consider or have reason to consider) of our beliefs, take precedence over fallibility considerations. The operator thus behaves *quasi-strictly*, so to speak. Relative to ordinary contexts, it behaves like the corresponding *ought* would, thus preserving our Strictness-supporting intuitions. It is only in Preface-like contexts, that the defeasible nature of the deontic operator makes itself felt. For it is in these situations that our Preface intuitions trump our Strictness intuitions. In this way, our Preface and our Strictness intuitions are reconciled.

Needless to say, this is but a sketch. However, in part at least, the account's sketchiness is owed to the very nature of the phenomenon. There are no sharp boundaries demarcating ordinary from Preface-type contexts. But the vagueness in question need not deter us. After all, our aim was to identify the characteristic features of Harman's account of epistemic norms and to show that there is room for a logical bridge principle within such an account. Our preliminary conclusion, then, is that even when we pose the question of the normativity of logic in Harman's own terms, we can provide an affirmative answer. We have formulated a qualitative guiding bridge principle (in place of IMP) that is immune not only to Harman's objections, but also to MacFarlane's additional desiderata. The bridge principle we ended up with, $Wr+a^*$, strikes me as both substantive and correct given its normative role. Namely, the role of instructing the deliberating agent as to how she should go about managing her beliefs in light of what she takes to follow from what.

To some our conclusion may come as a surprise. Harman's skepticism vis-à-vis NT is often understood as a flat out denial of the very possibility that any bridge principle should meet the mark. However, I believe Harman's position on the question of NT is rather more nuanced than it is often made out to be. While Harman is correctly described as denying that deductive logic is, in and of itself, a 'normative theory'. He leaves open the possibility that implications and inconsistencies that are readily recognizable to the agent—what he calls 'immediate implication' and 'immediate inconsistency'—are 'important for reasoning' (Harman 1986, p. 20).³¹ The bridge principle we have identified seems to bear this out in more precise terms, and so does not seem to be at odds with what Harman says.

³¹See also (Harman 2009).

How, then, does Harman arrive at the conclusion that there is no ‘significant way’ in which logic is ‘*special*ly relevant to reasoning’ (Harman 2009, p. 334, Harman’s emphasis)? Harman’s explanation is given in the following passage.

It would seem one can recognize a logical implication or logical inconsistency only if one has the relevant concept of logical implication or logical inconsistency. But it would seem that few people have such concepts, at least if this involves distinguishing logical implication and inconsistency from other sorts of implication and inconsistency. (Harman 1986, p. 17).

Ordinary folk do not, for instance, distinguish implications that hold in virtue of logical form from other types of necessary entailments like ‘If Philadelphia is south of New York City, then New York City is north of Philadelphia’ or that if ‘ A is a part of B and B is a part of C , then A is a part of C ’ and others like it. It is for this reason that logic does not, according to Harman, have any *special* normative role in reasoning. Some immediately recognizable logical relations and properties may indeed be of relevance to reasoning, but their properly *formal logical* character does not set them apart from any old non-logical analytic or material implication. Whence, the lack of any *special* importance of logic for reasoning.

Now, I readily grant that the problem of providing a principled demarcation of what William Kneale once called ‘the province of logic’ is a notoriously thorny question. Furthermore, even if a characteristic property of logic were to be made out—perhaps its topic-neutrality understood as invariance of logic’s operations under all permutations of the objects in the domain—it would still need to be shown that this distinguishing feature bestows upon logic a privileged role in the reasoning of ordinary agents, setting it apart from non-logical implications. If that is what it takes to establish logic’s ‘special relevance’ for reasoning, it is indeed a tall order.

It is just not clear to me, however, what hangs on the qualification ‘special’ here. The question with which we began was whether logic has a normative role to play in our epistemic lives. More precisely, we asked whether it can play the *guiding* role that most interests Harman when it comes to epistemic norms. I submit that our discussion has shown that it can, and W_{r+a}^* states exactly in what sense.

7 Evaluative Bridge Principles

We have seen that there are several ways of disambiguating NT. For Harman the normative role of logic, if any, must reside in the way in which it yields prescriptions as to how, from the perspective of subjective first-person deliberation, we should

reason given what we believe about the logical relations in which our belief contents stand. I now want consider a different interpretation, namely the interpretation according to which the role of logic is above all *evaluative*. A bridge principle so understood is a device that classifies, from an external third-person perspective, a belief states into ones that are and ones that are not logically intact, in the sense that the logical consequences of the propositions believed are adequately taken into account. What might such a principle look like?

One of the characteristic features of evaluative principles, as opposed to guiding ones, is that they need not be attitudinally constrained—the Excessive Demands Objection, as we have seen, is inapplicable when it comes to them. This is because the *ought* involved in such principles is an evaluative *ought*, which does not presuppose the ability on the part of the agent to bring about a favorably classified doxastic state.³² More generally, we expect a principle whose purpose it is to display how, objectively speaking, the logical consequences of the propositions believed should be taken into account to be a good deal more demanding than guiding principles. In particular, we would expect an evaluative principle to satisfy the Priority Question as well as the Strictness Test.

What about Clutter Avoidance? If all an evaluative principle is in the business of providing is an objective evaluation of a doxastic state (or proper subsets thereof) quite independently of the agent’s cognitive aims and limitations, then Clutter Avoidance ought not to enter the picture. If that is right, evaluative principles should not be restricted to those logical consequences the agent happens to consider or has reasons to consider: From a strictly evaluative standpoint all logical consequences matter. This feature draws our attention to a fact that should already have been apparent, namely that no ordinary agent stands any chance of qualifying as perfectly logically coherent by the standards of an evaluative bridge principle. However, the unattainability of an evaluative standard does not impugn its role as an ideal. An ideal we may seek, partially and locally, to approximate, and which thus yields standards for our directives and appraisals.

We have already encountered a principle above, $Wo+a^*$, which we might try to modify to suit our present needs. Stripping it of its unnecessary restrictions we obtain:

- $(Wo+)$ If $P_1, \dots, P_n \models Q$, then S *ought* to (believe all of the P_i only if S believes Q).

Being a wide scope principle it avoids the Too Much Instruction Objection and, *modulo* our previous remarks concerning the logical form of deontic operators, it also avoids Broome’s Objection. Moreover, it ticks all of the remaining boxes just mentioned, as can be easily verified. All the boxes, that is, *except* the Preface

³²The same goes, *mutatis mutandis*, for the other deontic modals.

Paradox challenge. Supposing, as we did, that our Preface intuitions are to be taken seriously, Wo+ fails us. In order to deal with the Preface Paradox, something's got to give. In the previous section our strategy was to fall back on the weaker *reasons* operator. We could adopt the same strategy here, leading us to the principle:

- (Wr+) If $P_1, \dots, P_n \models Q$, then S has *reasons* to (believe all of the P_i only if S believes Q).

However, as we noted, giving up Strictness may be tolerable when it comes to subjective guiding principles, but when it comes to evaluative principles, it would seem to be a non-negotiable feature. To see this consider again a Preface Paradox scenario. We had previously appealed to the *reasons* operator in such a case in order to account for the fact that the normative force of logic can be trumped in such cases. However, even if we admit that an author may be epistemically rational in believing each of the propositions making up the body of a book, the author's doxastic state will still be logically defective. An evaluative principle should presumably bear this out.

Given, then, that evaluative principles must respect Strictness, we face the following question: Can we have a bridge principle featuring strict deontic operators, which is able to reconcile both the intuitions driving Strictness *and* those driving the Preface Paradox? We can. But, as we noted above, we can only *provided* we go in for a quantitative bridge principle. Something would have to give in reconciling Strictness and the Preface Paradox challenge, we said. Here it is the notion of full belief that has to go.

As we mentioned on several occasions, Field has proposed such a bridge principle along these lines. We can state it thus:³³

- (DB) If $P_1, \dots, P_n \models Q$, then S 's degrees of belief ought to be such that:

$$cr(Q) \geq \sum_{1 \leq i \leq n} cr(P_i) - (n - 1)$$

³³I am skirting over some of the most intriguing and original features of Field's account. In particular, I am setting aside his distinction between 'projectivist' and 'realist' versions of bridge principles. All of the principles we have considered thus far were 'realist' principles in the sense that they are accompanied with a tacit presupposition that there is but one objectively correct notion of logical consequence—presumably classical consequence—and that this privileged notion of consequence is the source of any normative force logic might have. 'Projectivist' principles, by contrast, extend a form of non-factualism about epistemic norms (Field 2009c) to the domain of logical norms. Logic-induced norms stem not from any mind-independent facts about entailment, but rather from the agent's commitment to a particular logic. Bridge principles thus take the form 'If one regards argument $P_1, \dots, P_n \models Q$ as valid...'. Nothing at present hinges on this issue, I will therefore set it aside.

Note that DB is a wide scope principle: It requires that our degrees of belief respect the specified inequality, which can be achieved either by adjusting one’s degrees of belief in the conclusion or by adjusting one’s subjective probabilities in the premises. DB amounts to a ‘hybrid’ approach: It is a *logical* bridge principle—it articulates the ways in which we ought to manage our beliefs as directly consequent upon the logical relations in which their contents stand without a detour via probability theory; at the same time, it is a *quantitative* principle that relates logical norms to degrees of belief, not to full beliefs.

DB is based on a well-known result in probability logic, which is usually stated in terms of ‘uncertainties’.³⁴ Define the *uncertainty* of a proposition P , $u(P)$ as $u(P) = 1 - cr(P)$. Put in this way, DB says that the uncertainty of the conclusion must be less than or equal to the sum of the uncertainties of the premises. DB can be seen to share a number of important features with standard probability theory. Plug in 0 for n and you get that one should assign 1 to any logical truth. Plug in 1 and you get that one’s degree of belief in the premise of a valid single-premise argument should not exceed your degree of belief in the conclusion. The idea underlying DB is that uncertainties can add up and therefore need to be accounted for when we are trying to determine how the logical relations between our belief contents should affect our degrees of belief in those contents. Even if my uncertainty about each of a large number of premises is next to negligible when taken individually, the uncertainty may accumulate so as to make the conclusion highly (perhaps even maximally) uncertain. More precisely, if each premise of a multi-premise argument has uncertainty of at most ε , then if the argument has at least $1/\varepsilon$ many premises, the conclusion will be maximally uncertain. It is due to this feature that DB (unlike its qualitative cousins) is able to account for the epistemic paradoxes. In particular, it can account for the Preface Paradox. If I believe each of the propositions in my book to degree (say) .99, DB requires that my degree of belief in their conjunction (call it R) lie between 0 and $\min(cr(P_i))$ (where the P_i are the claims in the book). Hence, in our case, $0 \leq cr(P_i) \leq .99$. Thus DB is compatible with $cr(R)$ ’s being very low, even 0.

Let us explore Field’s principle some more. One worry one might have is that DB is too weak; that it fails to capture all of the constraints we naturally would expect logic to impose on our degrees of belief. The following simple example may serve as an illustration of this type of worry. Suppose I have middling degree of confidence in both the propositions P and Q , i.e. my credence in both of them is 0.5. Suppose further that P and Q jointly entail R . DB now places the following demand on us: $cr(R) \geq cr(P) + cr(Q) - 1$. It thus requires that my belief in R be at least of degree 0. But is this not just to say that according to DB the fact of R ’s being logically entailed by the P and Q imposes *no constraint whatsoever*

³⁴See (Adams 1998) for more details. For a helpful overview, see (Hájek 2001).

on the degree of belief I should have in R (given my degree of confidence in the premises)?

In fact, this result is as unsurprising as it is unproblematic: The uncertainty associated with the two premises may add up to make the conclusion maximally uncertain. And in some cases this seems exactly right. For instance, in case of a logic that comprises

$$\text{(EXP) } P, \neg P \models Q$$

this just what we want. Since a contradictory pair of premises entails any proposition whatsoever, and since some propositions ought to be maximally disbelieved, a minimal lower bound on $cr(Q)$ is desirable.

However, suppose now that R happens to be of the form $P \vee Q$ (for simplicity, let ‘ \models ’ denote the classical consequence relation), it then seems reasonable to expect a more informative lower bound on $cr(P \vee Q)$. Presumably the fact that $P, Q \models P \vee Q$ should translate into a stronger constraint (given our middling credence in the premises) than the trivial constraint that $cr(P \vee Q)$ lie somewhere (anywhere) in the unit interval. What we would expect is a constraint to the effect that $cr(P \vee Q)$ be no less than the credence invested in either of the disjuncts. Is such a constraint to be had on Field’s account? In fact, it follows straightforwardly from DB itself. Assuming the logic we are considering includes \vee -introduction, DB requires that $cr(P \vee Q) \geq cr(P)$ and $cr(P \vee Q) \geq cr(Q)$. But that is just to say that $cr(P \vee Q) \geq \max(cr(P), cr(Q))$.

Hence, we can derive the desired lower bounds in the single-premise case. Notice, however, that applied directly to $P, Q \models P \vee Q$, DB still yields the trivial lower bound of 0. This points to a general (if minor) problem in Field’s account. Adding further premises to a valid argument may diminish the lower bound on the degrees of belief one ought to have towards the conclusion—for instance, in adding an additional premise one, as it were, ‘lessens’ the normative force a valid argument exerts on one’s credences. In our example, introducing an additional premise Q (or indeed any other premise to which we attach a middling degree of belief), yields the result that DB issues in no constraint at all for $cr(P \vee Q)$. The method generalizes, of course: Given the structure of DB we can ‘weaken’ any valid argument by adjoining extra premises (that are less than certain) to it, thereby ‘lessening’ the normative force of the original argument at will. Indeed, by adding the appropriate number of premises, we can always bring the lower bound on the prescribed degrees of belief of the conclusion down to zero.³⁵

³⁵This is a straightforward consequence of Adams’ aforementioned observation, see (Adams, op. cit., p. 39).

But this seems wrong. Not catastrophically wrong since DB still yields $cr(P \vee Q) \geq \max(cr(P), cr(Q))$ in a roundabout way, but it seems odd all the same. The normative impact a valid argument has on my system of credences should be unaffected by the adjunction of further premises to it. At least this should be so in the case of logics with monotonic consequence relations.

But perhaps that is just it. Field aims to formulate DB in such a way as to be compatible with a wide range with logics, in particular logics whose arguments' validity is not generally preserved under the adjunction of extra premises. Perhaps then the awkward feature we are presently discussing is just the cost for DB's wide range of applications to non-classical logics?

This explanation will not cut it. For take the case of, say, relevant logics in which the rule of weakening ($S \models Q/S, P \models Q$) is not generally valid. If a valid argument really does exert normative force on my system of credences, the introduction of irrelevant premises into a valid argument does not somehow lessen the force it previously had on me. According to the relevant logician, the introduction of irrelevant additional premises should render the argument invalid and so annul any normative force it may have had altogether. But that is not the result we get with DB.

What, then, can be done to rid Field's proposal of this irksome bug? The answer to the problem is, I think, to be found in Adams (*op. cit.*). The thought is this. Going back to our example, we note that the addition of Q in $P, Q \models P \vee Q$ is superfluous, because Q is inessential to the validity of the argument. Call a premise (in a valid argument) *essential* if omitting it would transform a valid argument into an invalid one. Moreover, call a set of essential premises *minimally essential* if it has no proper subsets that are essential. The idea now is that the uncertainty of inessential premises should be discounted when determining the uncertainty of the conclusion. More precisely, Adams has us weight the uncertainty of each premise by its 'degree of essentialness', $e(\cdot)$. If, in an argument $P, P_1, \dots, P_n \models Q$, P does not belong to any minimal essential premise set, then its degree of essentialness $e(P)$ is 0. If it does, we determine its degree of essentialness as follows. We consider the cardinality of the smallest minimally essential premise set to which it belongs and take its reciprocal. That is, if the cardinality of the set in question is (say) k , we have $e(P) = 1/k$. Putting these notions together, we arrive at the following inequality:

- (DB') If $P_1, \dots, P_n \models Q$, then S 's degrees of belief ought to be such that $u(Q) \leq \sum_{1 \leq i \leq n} e(P_i)u(P_i)$

Does this help? Consider the argument $P, R \models P \vee Q$. Clearly, the degree of essentialness of R , $e(R)$, is zero because R is inessential. We thus arrive at the desired result $u(P \vee Q) \leq u(P)$. The problem reoccurs, however, when we consider $P, Q \models P \vee Q$. Neither premise is essential in the presence of the other.

Nevertheless for either premise, the smallest essential premise set is the singleton containing the premise. Consequently, in the case where $cr(P) = cr(Q) = 0.5$, we again obtain the unwelcome result that the uncertainty of the conclusion is maximal (i.e. there is no lower bound on the credence we should have in the conclusion). DB' thus fails to deliver the optimal lower bound. What is more, the case of inconsistent premises poses similar problems. Take $P, \neg P \models Q$ once more. The smallest essential premise set would seem to be $\{P, \neg P\}$, we thus get: $u(Q) \leq 0.5$, which is clearly false. One might impose an inconsistency constraint on the premises of the argument. However, given our purposes this is no option for us. After all, a central motivation for going with a quantitative bridge principles linking logical principles with norms on credences was to accommodate Preface Paradox-like cases—cases, that is, in which one might rationally have a high degree of belief in each member of an inconsistent set of premises.

In light of these shortcomings of DB', I propose we stick with a modified version of Field's original principle DB. We can secure the desired lower bounds on our credences in the conclusions as follows. Given the argument, delete any premises that are part of no essential premise set. When confronted with arguments containing premises that are part of an essential premise set but which are redundant in the context of the argument, delete all such redundant premises except for the one with the highest subjective probability (i.e. lowest uncertainty). For example, in $P, Q \models P \vee Q$, where $cr(P) = 0.3$ and $cr(Q) = 0.8$, we retain Q as the only premise in order to ensure that $cr(P \vee Q) = \max(cr(P), cr(Q))$. In this way, we are able to ensure that the bounds on our degrees of belief in the conclusion is optimal.

Before we turn to our final class of bridge principles, appraising principles, there is a further issue that merits our attention—an issue pertaining to the particular nature of quantitative bridge principles. We have presented DB as a hybrid approach. Like probabilistic approaches to the epistemic paradoxes, the rational norms imposed by logic are related to degrees of belief rather than to full beliefs. However, unlike how probabilists would have it, the norms themselves are genuinely logical in character; degree of belief talk, on this view, need not invite probabilistic norms of rationality. Consequently, norms of logic do have a place in an account of theoretical rationality.

This raises a potential objection from the probabilist camp. The worry is that one's acceptance of DB and its ilk already smuggles in an implicit commitment to probabilism. For in order to determine whether, e.g. $cr(P \vee Q) \geq cr(P)$, we must be able to compute the numerical value of $cr(P \vee Q)$. To do so, where else would we look if not to subjective probability theory? Subjective probability theory, according to this line of thought, 'embeds logical norms' (Williams Forthcoming, p. 1). Christensen appears to have a similar idea in mind when he writes that

the constraints of probabilistic coherence ‘flow directly from the standard logical properties of the believed claims (idem, p. 15). For instance, the constraint that $cr(P \vee Q) \geq cr(P)$ might be thought to ‘flow directly’ from the fact that $P \models P \vee Q$. But if subjective probability theory embeds logical norms, while also providing us with the quantitative tools necessary to implement DB, it might look as if our hybrid bridge principle has become superfluous: When all is said and done, it is really probabilism that is doing all of the normative work and only it deserves a place in an account of epistemic rationality. Deductive logic is built into the probabilist machinery, but it carries no normative weight of its own.

Notice that this line of thought can be generalized to non-classical logics. As Field (Forthcoming) shows, DB is not tied to any particular logic. The principles might be thought to express the core normative properties common to a broad class of logics (e.g., relative to one’s logic L , one ought to have credence 1 in any L -tautology; one’s degree of belief in the single premise of a L -valid argument ought not exceed one’s degree of belief in the conclusion, and so on.) So far, then, this appears to show that classical probabilism is not implicit in or entailed by DB’ after all. For take an advocate of intuitionistic logic. Clearly, it would be irrational for such an agent to give credence 1 to every instance of $P \vee \neg P$. Equally clearly, it would be a mistake to evaluate the credence set of such an agent according to classical probability theory.

But here, now, is how these considerations can be seen to fit with the probabilist line of thought offered in the next to last paragraph. What our invocation of non-classical logics really shows is that while DB does not commit us to *classical* probabilism, it can still be claimed to commit us to what we might call *generalized probabilism*. The idea is this. Following Field, interpret ‘subjective probability theory’ liberally to mean ‘theory of acceptable combinations of degrees of belief’ (as opposed to ‘classical theory of subjective probability’) (Field Forthcoming, p. 16). Generalized probabilism then demands that we be probabilistically coherent, but without presupposing allegiance to any particular logic. That is, it demands probabilistic coherence with respect to the probability theory based on one’s preferred logic.³⁶ In this way we again seem to be led to the conclusion that it is (generalized) probabilism that shoulders the entire normative burden, and that logical norms again take the back seat.

I do not wish to pass judgment on the merits of probabilism here.³⁷ It is true that an evaluation of an agent’s system of credences by means of DB requires some system of rules instructing us how to perform the necessary calculations. And it

³⁶See (Williams 2012) for a helpful survey of non-classical probability theories, and (Williams forthcoming) for a survey of ways in which standard justificatory results like Dutch-book and accuracy-dominance arguments can be extended to non-classical probability theories.

³⁷See (Hájek 2008) for an illuminating discussion of the standard arguments in favor of probabilism.

may be that probability theory is the most ‘natural’ (whatever that means) such system. The claim I want to make is this: Even if DB is naturally accompanied by probabilism, it does not entail it, nor is it the case that logical evaluative norms are rendered superfluous by probabilism.

As for the first part of my claim, it suffices to observe, that the connections between deductive logic and probability theory are not as tight as Christensen makes them out to be. Probability theories generally impose stronger requirements than those imposed by DB. DB does not, for example, require anything like Additivity (i.e. in the sense of standard probability theory: If $\models \neg(A \wedge B)$, then $cr(A \vee B) = cr(A) + cr(B)$). As Michael Titelbaum points out:

Given any set of inequalities satisfiable by a probability function, one can construct a distinct function that satisfies those inequalities, Non-Negativity, and Normality, and yet fails to satisfy Additivity—just take the probability function and square each of its values. Whether or not logic gains its normative purchase on epistemic states by way of the probability axioms, norms requiring graded beliefs to conform to additive probability measures must be more than just applications of deductive relations (Titelbaum 2008, p. 678)

The point can be generalized when we consider generalizations of probability measures—what Joseph Halpern has called *plausibility measures* (Halpern 2003). The details are of no moment. What matters is that there are numerous such plausibility measures that fail to be probabilistic, but which satisfy the constraints imposed by DB.

These considerations suggest that DB does have a substantive evaluative role to play after all. It tells us how logic constrains our degree of belief. To be sure, subjective probability theories offers one attractive way of supplementing these logic-induced norms, but it does not supplant them.

8 Conclusion

Let me recapitulate. My point of departure was NT: The thesis that logic has a normative role to play in our epistemic lives. I reviewed Harman’s influential objections to NT thereby laying down criteria that a successful defense of NT would have to meet. I also presented and developed MacFarlane’s classification of bridge principles and explained how his project can be thought of as a systematic attempt at a response to Harman’s objections. However, I then went on to argue that before we could hope to make any progress on the question of the normativity of logic it would be necessary to clarify and indeed disambiguate it. I did this by showing that there are at least three distinct broadly normative roles logic might be thought

to occupy and hence three senses in which logic might be taken to be normative: It might be thought of as a purveyor of directives, of evaluations or of appraisals. Finally, I proposed precise formulations for each of the corresponding questions “Is logic normative in the sense of issuing directives/evaluations/appraisals?”. Bringing our extended framework of bridge principles to bear on these questions, I examined the first two questions. I argued that logic is normative in an interesting and substantive way both in the guiding and in the evaluative sense. I have not here examined the remaining third question as to whether logic is normative in the sense of affording appraisals. Appraisals, as we have seen, can be thought of as something akin to a hybrid between directives and evaluations. They are akin to directives in that they take the agent’s ‘perspective’ into consideration when assessing her. They are like evaluations in that they deal in third-person assessments as well as in the sense of involving a certain amount of idealization. There are good reasons for being hopeful that the resources appealed to in our response to the first two questions, might successfully be brought to bear on the last question, and that it too can receive a positive answer. However, I leave the working out of these ideas to future work.

References

- E. Adams. *A primer of probability logic*. CSLI Publications, Stanford, CA, 1998.
- W. Alston. The deontological conception of epistemic justification. *Philosophical perspectives*, 2:115–152, 1988.
- J. Broome. Normative requirements. In J. Dancy, editor, *Normativity*. Oxford University Press, Oxford, 2000.
- J. Broome. Rationality. In T. O’Connor and Sandis C., editors, *A companion to the philosophy of action*, pages 285–292. Blackwell, 2012.
- J. Broome. *Rationality through reasoning*. Wiley-Blackwell, Oxford, 2013.
- R. Carnap. *Logical foundations of probability*. The University of Chicago Press, Chicago, 2 edition, 1962.
- C. Cherniak. *Minimal rationality*. M.I.T. Press, Cambridge, MA., 1986.
- D. Christensen. *Putting logic in its place*. Oxford University Press, Oxford, 2004.
- K. Easwaran and B. Fitelson. Accuracy, coherence, and evidence. In T. Szabo Gendler and J. Hawthorne, editors, *Oxford studies in metaethics in epistemology*, volume 5. 2013. Forthcoming.

- R. Feldman and E. Conee. Evidentialism. *Philosophical studies*, 48:15–34, 1985.
- H. Field. What is the normative role of logic? *Proceedings of the Aristotelian Society*, 83:251–268, 2009a.
- H. Field. Pluralism in logic. *Review of symbolic logic*, 2:342–359, 2009b.
- H. Field. Epistemology without metaphysics. *Philosophical studies*, 143:249–290, 2009c.
- H. Field. What is logical validity? In C. Caret and O. Hjortland, editors, *Foundations of logical consequence*. Oxford University Press, Forthcoming.
- G. Frege. *Grundgesetze der Arithmetik*. Mentis, Paderborn, 1893/1903/2009.
- P. Geach. Whatever happened to deontic logic? *Philosophia*, 11:1–12, 1982.
- A. Goldman. *Epistemology and cognition*. Harvard University Press, Cambridge, MA., 1986.
- A. Goldman. Internalism, externalism and the architecture of justification. *Journal of philosophy*, 106:309–338, 2009.
- A. Hájek. Probability, logic, and probability logic. In L. Goble, editor, *The Blackwell Companion to logic*, pages 362–384. Blackwell, Oxford, 2001.
- A. Hájek. Arguments for—or against—probabilism? *The British journal for the philosophy of science*, 59:793–819, 2008.
- J. Halpern. *Reasoning about uncertainty*. M.I.T. Press, Cambridge, MA., 2003.
- G. Harman. *Thought*. Princeton University Press, Princeton, 1973.
- G. Harman. Logic and reasoning. *Synthese*, 60:107–127, 1984.
- G. Harman. *Change in view: Principles of reasoning*. M.I.T. Press, Cambridge, 1986.
- G. Harman. General foundations versus rational insight. *Philosophy and phenomenological research*, 63:657–663, 2001.
- G. Harman. Internal critique: A logic is not a theory of reasoning and theory of reasoning is not a logic. In *Handbook of the logic of argument and inference: The turn towards the practical*, volume 1 of *Studies in logic and practical reasoning*, pages 171–186. Elsevier, Amsterdam, 2002.

- G. Harman. Field on the normative role of logic. *Proceedings of the Aristotelian society*, 109:333–335, 2009.
- G. Harman. Epistemology as methodology. In J. Dancy, E. Sosa, and M. Steup, editors, *A Companion to Epistemology*, pages 152–156. Blackwell, Oxford, 2010.
- G. Harman and S. Kulkarni. The problem of induction. *Philosophy and phenomenological research*, 72:559–575, 2007.
- L. Humberstone. Two sorts of ‘ought’s. *Analysis*, 32:8–11, 1971.
- R. Jeffrey. Dracula meets wolfman: Acceptance vs. partial belief. In M. Swain, editor, *Induction, acceptance and rational belief*. Humanities Press, New York, 1970.
- N. Kolodny. How does coherence matter? *Proceedings of the Aristotelian society*, 107:229–263, 2007.
- N. Kolodny and J. MacFarlane. Ifs and oughts. *Journal of philosophy*, 107:115–143, 2010.
- J. MacFarlane. In what sense (if any) is logic normative for thought? Unpublished, 2004.
- J. MacFarlane. *Assessment sensitivity*. Oxford University Press, Oxford, 2014.
- D. C. Makinson. The paradox of the preface. *Analysis*, 25:205–207, 1965.
- P. Milne. What is the normative role of logic? *Proceedings of the Aristotelian Society*, 83:269–298, 2009.
- J. Murzi and F. Steinberger. Is logical knowledge dispositional? *Philosophical studies*, 166:165–183, 2013.
- S. Ryan. Doxastic compatibilism and the ethics of belief. *Philosophical studies*, 114:47–79, 2003.
- M. Schroeder. *Ought*, agents, and actions. *The philosophical review*, 120:1–41, 2011.
- W. Sellars. Language as thought and as communication. *Philosophy and phenomenological research*, 29:506–527, 1969.
- F. Steinberger. Frege and Carnap on the normativity of logic. *Synthese*, 2015. To appear in special issue ‘Carnap on logic’.

- F. Steinberger. Explosion and the normativity of logic. *Mind*, Forthcoming.
- M. Steup. Doxastic voluntarism and epistemic deontology. *Acta analytica*, 15: 25–56, 2000.
- M. Titelbaum. Review of *Putting logic in its place: Formal constraints of rational belief*. *Mind*, 117:677–681, 2008.
- B. Weatherson. Deontology and Descartes’ demon. *Journal of philosophy*, 105: 540–569, 2008.
- R. Wedgwood. Internalism explained. *Philosophy and phenomenological research*, 65:349–369, 2002.
- R. Wedgwood. The meaning of ‘ought’. In R. Shafer-Landau, editor, *Oxford studies in metaethics*, volume 1, pages 127–160. 2006.
- B. Williams. Deciding to believe. In *Problems of the self*, pages 136–151. Cambridge University Press, Cambridge, 1976.
- J. R. G. Williams. Generalized probabilism: Dutch books and accuracy domination. *Journal of philosophical logic*, 41:811–840, 2012.
- J. R. G. Williams. Accuracy, logic and degree of belief. In C. Caret and O. Hjortland, editors, *Foundations of logical consequence*. Oxford University Press, Oxford, Forthcoming. In preparation.
- J. R. G. Williams. Non-classical logic and probability. In A. Hájek and C. Hitchcock, editors, *Oxford handbook of probability and philosophy*. Oxford University Press, Oxford, forthcoming.
- T. Williamson. *Knowledge and its limits*. Oxford University Press, Oxford, 2000.