Harmony in a sequent setting: a reply to Tennant

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1 Introduction

In my Steinberger 2009 I argued that Neil Tennant's Harmony requirement is untenable because of its failure to account for the standard quantifier rules.[1] Instead of justifying the customary rules for the existential and universal quantifiers, Tennant's account appears to sanction only wholly unrestricted—and so patently disharmonious—quantifier rules. In his characteristically thoughtful response Tennant 2010, Tennant offers a sequent calculus version of his Harmony requirement that rules out such pathological would-be quantifiers. While I agree with Tennant that recasting his Harmony requirement in the sequent format as he proposes blocks the said disharmonious quantifier rules, I submit that Tennant's revamped Harmony requirement nevertheless misses the mark. I present two objections to substantiate my claim. First, I show that the crucial additional assumption underlying Tennant's sequent calculus-based account—what I call the admissibility assumption—is excessively strong: so strong, in fact, that it renders otiose the core of Tennant's original account. Second, I argue that the admissibility assumption, as a global requirement on deductive systems, is ill-suited for the purposes of codifying the intuitive notion of harmony. Fortunately, though, as I will demonstrate, we can dispense with the admissibility assumption altogether.

Tennant's strategy in his response, I had said, is to reformulate his requirement of Harmony in a sequent calculus setting. In order to distinguish the sequent calculus account from the original natural deduction-based account, let us refer to the two accounts as *S-Harmony* and *N-Harmony* respectively. Showing a (binary, say) logical constant $\$$ to be in S-Harmony involves, as in the case of N-Harmony, showing that the sentence $A\$B$ expresses (a) the strongest proposition[2] that may be derived by means of the introduction rules for $\$$; and (b) the weakest proposition derivable by means of the elimination rules for $\$$. In the sequent framework, a proposition *A* is said to be *logically stronger than* a proposition *B* just in case, from any sequent of the form $\Gamma, B : C$, one can derive $\Gamma, A : C$ but not conversely (Tennant 2010: 463). Tennant's Lemmas 1 and 2 demonstrate that the ordinary existential quantifier satisfies S-Harmony. It is crucial for our purposes to observe that the proof of Lemmas 1 and 2, like any demonstration of S-Harmony, *involve an essential appeal to the structural rule of CUT*.

At first blush it looks as if the demonstration of the S-Harmony of our customary, suitably restricted rules for the standard existential quantifier ∃ carries over to the rogue quantifier *E* whose elimination rule imposes no restrictions whatsoever on its parameter (Lemmas 3 and 4). But this conclusion, Tennant warns us, 'is over hasty' (idem: 465). The reason this is so, Tennant argues, is because *CUT* ought not to be '*taken as a primitive*

[1] I shall reserve 'Harmony' for Tennant's account of harmony. For the significance of the underlined capital 'H' and for further details of the account see (Tennant Forthcoming). I will use 'harmony' to designate the intuitive target notion that Harmony and other accounts like it are seeking to make precise.

[2] 'The proposition expressed by $A\$B$' is to be understood as the class of sentences logically equivalent to $A\$B$.

structural rule' (idem, Tennant's emphasis); the employment of *CUT* is not an automatic right, but has to be legitimated by a proof of its admissibility. Let us call this the *admissibility assumption*. Tennant's preferred proof of the admissibility of *CUT* is carried out in a natural deduction setting and amounts to an induction on the length of proofs and the complexity of the *CUT* formula. As such, it makes 'crucial use […] of the reduction procedures for the logical operators' (idem: 466). In the interest of terminological uniformity, let us follow Michael Dummett in calling the requirement that a pair of introduction and elimination rules ought to admit of a reduction procedure the requirement of *intrinsic harmony* (1991: 250). We can thus say that it is a precondition for the proof of the admissibility of *CUT* that all the logical operators be governed by intrinsically harmonious rules. But, and this is the crux of Tennant's argument, *E* violates intrinsic harmony: no reduction procedure like that for ∃ is available for *E* since such procedures crucially rely on the arbitrariness of the parameter, which the customary restrictions, absent in the case of *E*, are designed to secure. In short: S-Harmony presupposes the admissibility of *CUT*, which in turn presupposes intrinsic harmony for all pairs of inference rules in the system. Intrinsic harmony is thus effectively incorporated into S-Harmony (via the admissibility assumption), thereby ruling *E* out of bounds.

2 Objections

Tennant is certainly right that S-Harmony successfully blocks pathological quantifiers like *E*. The question is whether Tennant's revised account constitutes an acceptable principle of harmony. Before we get to that, though, let us ask whether it is appropriate to regard S-Harmony as a *revision* of Tennant's Harmony principle at all. Tennant's introductory remarks strongly suggest that he intends his paper as a defence of his *original* position: S-Harmony is not a modification of his Harmony requirement, but merely a way of presenting it in a different guise 'in order better to illustrate its application to the existential quantifier' (2010: 462). In other words, Tennant's claim seems to be that his original account already possesses the resources necessary for an adequate treatment of the quantifiers. Simply, S-Harmony, with its explicit statement of the *CUT* rule, brings these formerly merely implicit resources clearly into view. But this is surprising. As we have seen, the reason S-Harmony is able to rule out the aforementioned pathological quantifiers is because it incorporates the crucial admissibility assumption. If S-Harmony and N-Harmony are merely alternative modes of presentation, the same assumption (or at least a functionally equivalent assumption, e.g. normalizability) must already have been part and parcel of Tennant's original account. However, as far as I could make out none of Tennant's earlier presentations of the Harmony principle incorporate the admissibility assumption or any of its equivalents ((1978: 74-77), (1987: 94-98), (1997: 314-22), (2008)). Harmony and intrinsic harmony (which is often mentioned in conjunction with normalizability) are consistently presented as *independent* albeit compatible formal correlates of the intuitive notion of harmony.[3] It is thus not clear, *pace* Tennant, in what sense the admissibility assumption is already supposed to have been implicit in the original N-Harmony account.

Be that as it may, the important question is whether Harmony based on the admissibility assumption amounts to a good account of harmony. I want to argue that it does not. In what follows I will present two arguments to this effect. The first argument is this. If, in adopting the admissibility assumption, we build (in effect) intrinsic harmony into the very notion of Harmony, it is really intrinsic harmony (and not harmony, the core element of

[3] Indeed, Tennant goes so far as to conjecture that Harmony and intrinsic harmony are equivalent (Tennant Forthcoming: 15). But this cannot be quite right. The quantum-logical disjunction operator discussed below constitutes an example where the two accounts come apart (even in the presence of the admissibility assumption): 'quantum-or' satisfies intrinsic harmony, but violates Harmony (though it does satisfy harmony).

Tennant's notion of Harmony) that does all of the work. To see this, recall that harmony, intuitively understood, is the principle that the introduction and elimination rules governing the meaning of a given logical constant ought to be appropriately balanced: the elimination rules ought to give rise to *all* and *only* the deductive consequences that already follow from the premises of the corresponding introduction rules. Following my (2009: 655), we can distinguish two forms of disharmony: *E-strong* disharmony obtains when the strength of the elimination rules outruns that of the corresponding introduction rules (the infamous *tonk* and *E* both fall into this category); *E-weak* disharmony occurs when the elimination rules associated with a logical operator are too weak fully to exploit the deductive consequences that sentences containing the operator as their principal operator enjoy by virtue of the operator's introduction rules (imagine an operator, *plonk* say, governed by the standard introduction rule for conjunction and the elimination rule for disjunction). A major part of the problem with the admissibility assumption, as we will see, is that it is sufficient on its own to guard against E-strong disharmony. In other words, the admissibility assumption alone already performs the principal task that we want a harmony constraint to perform, namely to rule out *tonk*ish operators, and so puts Tennant's harmony requirement out of work.

Admittedly, though, the admissibility assumption (and hence *a fortiori* intrinsic harmony) cannot quite do *all* of the work. Harmony may still be serviceable when it comes to blocking the opposite vice of E-weak disharmony (*plonk* and other E-weakly disharmonious operators like quantum-logical disjunction readily admit of reduction procedures). To see how Tennant's account achieves this, we must remind ourselves that Harmony actually falls into two parts: harmony and the maximality principle. harmony requires (roughly) that the proposition expressed by *A$B* be the strongest deducible from its introduction rules and the weakest to figure as the major premise of the corresponding elimination rules. To promote the 'h' to a capital 'H', Tennant requires in addition that *$*-E be the strongest rule in harmony with *$*-I and, conversely, that *$*-I be the strongest rule in harmony with *$*-E. Let us call this additional requirement the *maximality principle*.[4] Hence, the equation we end up with is this:

(1) harmony plus the maximality principle guarantee Harmony.

Now, neither harmony nor intrinsic harmony offer a solution to the problem of E-weak disharmony; whence the need for the maximality principle. Unlike intrinsic harmony, however, harmony, when taken on its own (i.e. without the admissibility assumption), is impotent to deal even with E-strong disharmony—this is just what the counterexample involving *E* demonstrates. Tennant's solution to this, as we have seen, is to introduce the admissibility assumption. In his quest for a notion of Harmony that safeguards against both E-strong and E-weak disharmony, he thus in effect further extends our above equation to the following:

(2) harmony plus the maximality principle *plus* the admissibility assumption jointly guarantee Harmony.

The trouble with this, as we mentioned above, is that the admissibility assumption (along with the requirement of intrinsic harmony which it presupposes) takes care of E-strong disharmony all on its own: a pair of rules that enjoys the property of intrinsic harmony is guaranteed not to

[4]Tennant 1978, the earliest formulation, comprised only harmony, which turned out to be insufficient for harmony since it does not guarantee a unique choice of rules. To my knowledge Tennant first supplemented harmony with the maximality principle in his (1987: 94).

display E-strong disharmony.[5] But this means that once the admissibility assumption is introduced, Tennant's harmony requirement no longer has any role to play in our equation and is thus reduced to an idle wheel.

We do, of course, still need the maximality principle to prevent E-weak disharmony. But here too the appeal to harmony is quite unnecessary. For nothing prevents us from simply coupling the admissibility assumption directly to the maximality principle, thus in effect identifying harmony with the conjunction of the requirements of intrinsic harmony (to take care of E-strong disharmony) and the maximality principle (to block E-weak disharmony).[6] We thus arrive at the following equation:

(3) The maximality principle plus the admissibility assumption jointly guarantee Harmony.

Our conclusions so far can we summed up as follows. In order to dodge the problem of rogue quantifiers, Tennant had to resort to the admissibility assumption. However, once we accept the admissibility assumption, Tennant's harmony requirement looks like an unnecessary detour. Appealing to the admissibility assumption directly and conjoining it with the maximality principle, we arrive at the same result while dispensing with the harmony principle altogether.

This brings me to my second argument. The claim here, remember, is that the appeal to the admissibility assumption is—the foregoing argument notwithstanding—both illegitimate and unnecessary. Let me explain. The admissibility of *CUT*, like the related property of normalizability, is generally a *global* property of a deductive system. That is, whether or not *CUT* is admissible in a system depends on the specific configuration of the rules that make up the system. Let us briefly illustrate this point with an example. Quantum logicians reject the following distributivity law:[7]

DIST: $A \,\&\, (B \vee C) \vdash (A \,\&\, B) \vee (A \,\&\, C)$

In order to block *DIST*, quantum logicians restrict the disjunction elimination rule (i.e. in sequent calculus terms the left-hand side introduction rule) by disallowing collateral

[5]To see this, suppose there was a connective *\$* governed by E-weakly disharmonious rules, which nevertheless admitted of a reduction procedure. By E-weak disharmony there must be a sentence *D* that can be deduced from *B\$C* via *\$*-E, but that does not already follow directly from all of the premises sufficient, *modulo* *\$*-I, for *B\$C*. Assume for simplicity that *A* is such a premise; i.e. *B\$C* follows from *A* by an application of *\$*-I, but *D* does not directly follow from *A*. Starting from *A*, applying *\$*-I to get *B\$C*, and applying *\$*-E immediately afterwards to obtain *D*, we arrive at a proof of *D* from the assumption *A*. By intrinsic harmony we should now be able to eliminate the resulting *\$*-maximum by applying the appropriate reduction procedure. But such a reduction procedure cannot exist, for if it did we would have a direct proof of *D* from *A*, which, *ex hypothesi*, does not exist.

[6]Stephen Read (2000, 2010) appeals to an inconsistency-inducing operator *bullet* in an attempt to show that harmony ought not to be equated with intrinsic harmony. However, I am unconvinced by Read's argument. Admittedly, the logical inferentialist does not, at present, dispose of a general account of the notion of a permissible introduction rule. Devising such an account in a philosophically well-motivated way is no mean task. Nevertheless there is no reason to assume that such a project should not be feasible. The case of *bullet* merely demonstrates the urgent need for such an account. That being said, even if Read were right, this would only strengthen my case against the admissibility assumption.

[7]The philosophical *locus classicus* for this version of quantum logic is (Putnam1968).

assumptions in the minor premises (i.e. *Γ* and *Γ'* are *required to be empty* in the following rule schema):

$$\frac{\Gamma, A : C \qquad \Gamma', B : C}{\Gamma, \Gamma', A \vee B : C}$$

Let us denote the quantum-logical disjunction operator as defined by the ordinary right-hand side introduction rules and the restricted elimination rule by '∪'. It is not hard to verify that *CUT* is admissible in the system *S* ={&, ∪}. Having thus satisfied the admissibility requirement, it can also easily be checked that ∪ is harmonious. (However, as we have mentioned above, ∪ is not Harmonious since the maximality principle selects the stronger, unrestricted disjunction elimination rule over quantum-disjunction.) But now augment *S* to *S'* = {&, ∪, ∨} by throwing the ordinary disjunction operator into the mix. *CUT* is not admissible in *S'*. If it were, ∪ would collapse into ∨ and *DIST* would be provable for quantum disjunction.

By incorporating the admissibility of *CUT* into his Harmony requirement, Tennant thus—contrary to his protestations to the contrary (2010: 467)—*does* 'go global'. Whether or not a given logical operator is governed by S-Harmonious rules of inference may turn not only on the rules themselves, but also on the system as a whole. Harmony, in its sequent 'formulation', is thus a global requirement. But this seems wrong. To see why, consider once again the case of *tonk*. One way of explaining what goes wrong in the case of *tonk* is in system-relative terms: e.g., *tonk* induces rampant non-conservativeness when introduced into *any* consistent system. However, such global accounts seem to miss the point. The reason *tonk* is semantically defective has nothing to do with the make-up of the systems in which it inheres; the real issue is that the introduction and elimination rules that determine its deductive behaviour are wildly mismatched. The insight generalizes: whether or not a logical expression satisfies harmony should depend only on the meaning-giving rules of inference it obeys, and these rules should not depend on any other logical constants.[8] It follows that harmony should be understood as a relational property of pairs of inference rules (and by extension of the logical constants governed by them) and not as a property of deductive systems. Consequently, the admissibility of *CUT*, a property of deductive systems, is not a candidate for formalizing the intuitive notion of harmony.

I have argued that the admissibility assumption introduces an illegitimate global dimension into Tennant's Harmony account. But why is it unnecessary? It suffices to observe that the active ingredient in Tennant's S-Harmony-based argument against the unrestricted existential quantifier is *intrinsic harmony*. The sole purpose of the admissibility assumption is to make available intrinsic harmony. But if that is so, why not simply skip the middle man and assimilate harmony *tout court* with intrinsic harmony augmented by the maximality principle? As we have seen above, the two requirements, jointly, are up to the job: intrinsic harmony secures against E-strong disharmony; the maximality principle wards off E-weak disharmony. There is no more need, therefore, to wheel in global considerations in the form of the admissibility assumption, as there was for the harmony requirement. We thus arrive at the final equation:

(4) The requirement of intrinsic harmony and the maximality principle jointly make up the fomal correlate of the intuitive notion of harmony.

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[8] At least this will be the case if the principle of separability—the principle that there is no need, in the schematic statement of the meaning-determining rules for any given logical constant, to invoke any other logical constant—is upheld. Tennant explicitly endorses separability (1997:316-17).

An approach broadly along these lines seems to me to be correct. However, I think that accounts are available that are equally faithful to the intuitive notion of harmony, but which boast, among other advantages, greater compactness and precision (see e.g. my 2008).

Conclusion

To recapitulate, I have argued that Tennant's reply to my (2009) is unsuccessful. For one, the account of S-Harmony he proposes to block freakish quantifiers like *E* is excessively strong, rendering the core element of his original account—the harmony requirement—superfluous. Moreover, the central ingredient of S-Harmony, the admissibility assumption, is, as I have tried to show, both illegitimate and dispensable. If I am right, Tennant's revised account of S-Harmony fares no better than N-Harmony before it.[9]

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