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adverbs, logic of

a logical system or an interpretation thereof that admits of the formalization of natural language sentences involving adverbial modification. First-order logic (on its customary interpretation) does not offer a satisfactory treatment of adverbs. The problem is this:

(1) Rachel spoke.

is standardly formalized as "*Fa*". But now consider

(2) Rachel spoke eloquently.

It seems that "spoke eloquently" must be formalized as a single predicate wholly distinct from "spoke" and so receive a fresh predicate symbol, yielding "*Ga*". But this seems inadequate for two reasons. First, the formalization obfuscates the explicit relation between the two verb phrases on account of their common element, "spoke". Second, the logical form of these sentences should lay bare the entailment relations between them: namely that (2) intuitively entails (1).

Donald Davidson proposed to tackle these difficulties by postulating that action sentences involve a hidden variable that ranges over events and is bound by an existential quantifier. On this reading (1) ought to be paraphrased as

(3) There is an event x such that x is a speaking by Rachel.

Adverbs and adverbial clauses are thus treated as predicates of expressions that refer to events. Though cumbersome, this circumlocution promises to solve both of the aforementioned difficulties. For when we now paraphrase (2), we obtain

(4) There is an event x such that x is a speaking by Rachel and x is eloquent.

This brings to light both that (1) and (2) comprise a common element (the event of Rachel's speaking). Moreover, by representing the logical form of adverbially modified action sentences as existentially quantified conjunctions, Davidson ensures that the intuitive entailment relations obtain, and obtain as a matter of logic.

Davidson's account has been extremely influential, giving rise in particular to *event semantics*. All the same, it faces difficulties (of which Davidson was not unaware). For example, the account is unable to accommodate intensional adverbs (e.g.\ deliberately, greedily). Also, adverbs like "slowly" pose problem. The underlying reason for both types of difficulties is that any given action can be described in a number of ways.

The perhaps best-known subsequent development is the theory of adverbs proposed by Richmond Thomason and Robert Stalnaker. However, working in the tradition of Montague semantics, their aim is to provide a formal semantic treatment of adverbs; they do not seek to elaborate a *logic* of adverbs.

See also Davidson, Donald, logical form, Montague grammar.

dialetheism

A view chiefly promoted (in its modern form) by Graham Priest according to which there are *dialetheias*. A dialetheia is a proposition A, such that both it and its negation, $\neg A$, are both simultaneously true. Assuming that a proposition is false just in case its negation is true, dialeitheism amounts to the view that there are *some* true contradictions. Dialetheists thus reject the principle of contradiction. Relatedly, since, given explosion, a contradiction trivializes a system inasmuch as any proposition can be derived within it, dialetheists must reject Explosion. Dialetheism is often motivated by paradoxes of self-reference.

See also explosion, paradoxes of self-reference, principle of contradiction.

explosion

A principle of logic according to which an inconsistent set of propositions logically entails any proposition whatsoever. For example, from the contradictory pair of premises that aardvarks are and are not indigenous to Africa it follows that pigs can fly.

In classical logic, the validity of explosion flows directly from the definition of logical consequence as necessary truth-preservation in virtue of logical form. However, explosion is not valid in so-called *paraconsistent logics* like relevance logics and dialeithic logics.

See also dialetheism, *ex contradiction quodlibet**, *ex falso quodlibet**, relevance logic.

harmony, proof-theoretic

A requirement on inference rules that aims to rule out defective would-be logical constants, while ruling in well-behaved ones. Harmony plays a pivotal role in inferential approaches to the logical operators. According to such approaches, the meaning of a logical operator is determined by the inference rules that govern it. However, there are putative logical constants that are governed by inference rules that are individually irreproachable, but which jointly give rise to pseudo-constants with undesirable features. For example, Arthur Prior laid down the following pair of rules for the binary ‘connective’ *tonk*: *A / A tonk B* and *A tonk B / B*. Adding *tonk* to a standard logical system immediately sinks it into inconsistency.

What arguably goes wrong in the case of *tonk* is that its introduction and elimination rules are not appropriately balanced. The deductive strength of the elimination rule intuitively outruns that of the corresponding introduction rule: by introducing and subsequently eliminating *tonk* one is able to derive conclusions not otherwise warranted. Though less disastrous, the opposite vice---where the elimination rules are too weak for the corresponding introduction rules---should arguably also be proscribed. The goal of a harmony requirement, therefore, is to provide a philosophically well-motivated formal criterion that ensures that a constant's introduction and elimination rules are commensurate in deductive strength.

See also tonk.

Lockean thesis

a thesis introduced by name by Richard Foley concerning the relation between the qualitative notion of full belief and the quantitative notion of degree of belief (which we may think of as being represented by a function taking propositions to the unit interval). The Lockean thesis is a normative thesis that states that an epistemically rational agent believes a proposition *p* just in case the agent's degree of confidence in *p* exceeds a given threshold. Since it is implausible that we only believe propositions of whose truth we are certain, it is generally thought that the threshold in question may be less than one. Its value is thought to be determined by features of the context.

See also Bayesian rationality, belief, Locke, J.

logical pluralism

the view that there is no one true logic, but rather that there exist several conceptions of validity and logical consequence all of which are equally deserving of being called "logic". The question of logical pluralism arose with the advent of alternative logics such as intuitionist logic, quantum logic and paraconsistent logics. Advocates of such logics tended to be logical *monists* who maintained that their preferred logic should supplant classical logic. It is important to be clear about the sense in which monists oppose pluralism. Logics can be viewed purely as abstract mathematical structures whose properties may be studied in their own right. Alternatively, logics may serve as instruments for modelling certain phenomena, electric circuits or grammatical categories, for example. "Pluralisms" of these kinds are innocuous. The issue of pluralism arises in a meaningful way only when we consider logic as an account of what follows from what and so of which arguments are valid. Pluralists may agree that a valid argument is one such that in any case in which the premises are true, the conclusion holds also. Pluralism kicks in by allowing multiple non-equivalent but equally legitimate reinterpretations of "case". For example, plugging in "possible world" for "case" we obtain a version of classical consequence; interpreting "case" as incomplete but consistent "states of knowledge" one arrives at intuitionist logic, etc. Pluralism is reminiscent of, but distinct from, Carnap's "principle of tolerance" according to which there are several equally acceptable linguistic frameworks.

See also Carnap, R., intuitionist logic, philosophy of logic, quantum logic.

normalization theorem

a theorem of structural proof theory for natural deduction systems stating that any derivation in a given natural deduction system can be converted into a particular normal form. Like the cut-elimination theorem for sequent calculi, the normalization theorem has its source in

Gerhard Gentzen's *Hauptsatz* and was developed by Dag Prawitz and others. More precisely, the theorem states that any derivation $S \mid\!- C$ in a given system can be transformed into a derivation with the same premises and conclusion, which does not invoke any "extraneous formulas", i.e. formulas that are neither subformulas of one of the premises or of the conclusion. Intuitively, a derivation in normal form is one in which the premises are first dismantled by a series of applications of elimination rules and then progressively reconstituted by applications of introduction rules so as to create a direct deductive path from premises to the conclusion. Normalization is closely related to beta-reduction in typed lambda calculi via the Curry-Howard isomorphism.

See also cut-elimination theorem, deduction, lambda calculus, proof theory.

preface paradox

an epistemic paradox originally due to David Makinson thought by some to afford an example of a situation in which an agent is rationally justified in holding a set of beliefs she knows to be inconsistent. The paradox is generated as follows. An author composes a non-fiction book. Seeing that the claims in her book are the product of meticulous research, she has strong grounds for believing each of the n propositions that make up the body of her book: $p_1,\ldots,p_n$. The author also has overwhelming inductive evidence for believing that she, like any author who embarks on a comparable non-trivial project, is bound to be mistaken about some of the claims in her book. She expresses this belief in her own fallibility in the preface of her book in the form of a standard clause to the effect that she assumes responsibility for the errors that will inevitably be discovered in her book. Call the proposition that the book is bound to contain errors q. Clearly, q is equivalent to $\neg p_1 v \ldots v \neg p_n$. But then $p_1,\ldots,p_n$ and q cannot be jointly true. So, our author has an inconsistent set of beliefs.

See also justification, lottery paradox.

proof

a demonstration of the truth of a proposition. In one ordinary use of the term, a proof is an event the occurrence of which establishes the truth of a claim. For instance, someone may prove she can solve an equation by doing it. Sometimes "proof" is also used to refer to a piece of evidence in support of the truth of a proposition, as when the blue paw prints are taken to be proof of the dog's having eaten the blueberry pie. Relatedly, people sometimes speak of "scientific proof" adverting to inductive arguments in which the premises render the conclusion likely without guaranteeing its truth.

In its principal, stricter sense, however, a proof is a conclusive deductive argument. However, not any necessarily truth-preserving argument qualifies as a proof. For instance, the argument "*x, y, z* are natural numbers and n is a natural number greater than 2; therefore, $x^n+y^n \neq z^n$" has been shown to be necessarily truth-preserving, but hardly constitutes a proof of Fermat's last theorem. What this shows is that proofs have an essentially epistemic character. The deductive path leading from (perhaps only provisionally) accepted premises to the conclusion must be broken down into inferential steps of "manageable size".

This is not to say that the sole purpose of proof is epistemological. By demonstrating which conclusions rests on which assumptions, proofs may be seen to give us insight into the "order of things”. This role of proof is central to Fregean logicism, which aims to show that an arithmetical truth can be traced back to elementary laws of logic (plus definitional extensions). More recently, proof theory is deployed in order to investigate, what assumptions or axioms are needed in order to prove a given theorem.

This illustrates that proofs are not merely certificates of the truth of a proposition. Mathematicians may expend considerable energy in seeking novel proofs for theorems that have already been proved. The motivations are multiple. Some proofs may be more explanatory or informative than others, for example in that they enable one to see not only *that* a theorem holds, but also *why* it holds, or because they are constructive. Also, some proofs may be of greater theoretical significance, because they forge unforeseen connections between different fields of mathematics. Moreover, some proofs may be of greater theoretical value, because they contain or constitute techniques that admit of wider applications. Certain proofs may be deemed preferable on account of dispensing with contentious assumptions (e.g. proofs that do not rely on the axiom of choice). In the 20th century, computer-assisted proofs (e.g. of the Four-color theorem) have invited controversy among philosophers and mathematicians alike.

With Euclid’s *Elements*, the *axiomatic method* has become a model for the systematization of mathematical knowledge. Since then, and especially in the 19th century, the demand for rigor in proofs has increased dramatically, culminating in the notion of a fully formalized proof. These innovations have lead to greater perspicuity as well as paving the way for new areas of research, in particular for proof theory in which proofs themselves are taken to be the objects of mathematical inquiry. That being said, the vast majority of mathematical proofs that are published are informal (though rigorous). Informal proofs, it is usually assumed, could in principle serve as sketches, which, given sufficient patience and time, could be converted into fully formal proofs.

See also axiomatic method, formalization, proof theory.

principal principle

a principle proposed by David Lewis stating that a rational agent’s degrees of belief accord with single-case chances as follows: let c be a probabilistically coherent credence function. Let A be a proposition, t a time and X the proposition that the chance of A being true at t is x. Finally, let E be a proposition compatible with X and admissible at t. The principle then states that:

$$c(A|XE)=x$$

Much turns on the notion of X’s admissibility. An example will convey the general idea: suppose A states that a fair coin will land heads at time t. X says that the chance of A being true is 50%. But if E contains information about the coin toss’s actual outcome it would be inadmissible, because if the agent is aware of the actual outcome her credence in A should be much higher than 50%.

See also Bayesian rationality, Lewis, David, probability.

probabilism

the view that a rational agent's degrees of belief ought to be probabilistically coherent at any given time, i.e. that the function measuring the agent's credences is (or is extendable to) a probability function. Both probabilists and Bayesians endorse the (synchronic) norm of probabilistic coherence. However, unlike the Bayesian, the probabilist may not espouse the further diachronic norm of Bayesian conditionalization, i.e. that a rational agent ought to revise her beliefs by conditionalizing on her evidence.

Probabilism has traditionally been motivated by a number of arguments—the Dutch book argument, being the best-known among them—demonstrating that an expected utility maximizer who fails to conform her credence function to the axioms of probability theory, is less well off than she might have been (e.g. she faces a situation of certain monetary loss). Such arguments have been criticized for tying probabilism too closely to prudential rationality and thus lacking epistemological relevance. Recently, more properly epistemic arguments have been advanced such as "calibration" and "accuracy-dominance" arguments.

See also Bayesian rationality, Bayes' theorem, Dutch book argument, probability.

radical interpretation

an hypothetical procedure by which the linguistic behavior of a speaker is being interpreted without recourse to prior knowledge of her beliefs or the language she speaks. Donald Davidson, drawing on work by Quine, proposed radical interpretation as a basis of his truth-conditional theory of meaning. The problem, according to Davidson, is that it is not possible to assign meanings the speaker's utterances without knowing what she believes. The radical interpreter must therefore simultaneously assign meaning and ascribe beliefs in ways that are consistent with the speaker's behavior given the environment. In order to do so, the interpreter assumes the principle of charity. That is, the speaker is assumed to have generally true beliefs (by the interpreter's lights) thereby in effect enabling her to project her own beliefs on the speaker so as to get the process of interpretation off the ground.

See also Davidson, Donald, meaning, truth-conditional theory of, Quine, W.V.O.

regulative-constitutive distinction

a distinction going back to Kant popularized in a modified sense by John Searle. A rule or norm is said to be *regulative* if the practice or form of behavior it governs exists independently of the rule or norm. By contrast, a rule or norm is *constitutive* if it creates the very possibility of the existence of the practice or form of behavior. For example, traffic laws are regulative because they regulate a practice that could exist independently of any such rule—one can systematically violate the traffic code and still count as driving. The rules of chess, on the other hand, are constitutive of the game of chess because an agent not normatively bound by these rules would not count as playing chess.

See also Kant, I., Searle, J. R.

substructural logics

a family of non-classical logics weaker than classical logic. Substructural logics are characterized by the absence of certain *structural rules*. Whereas *operational rules* are rules in a formal deductive system that pertain to the deductive behavior of specific logical constants, structural rules do not mention logical operators, but rather encapsulate general properties of the deducibility relation. (Note that the distinction is not explicit in all types of proof systems.)

For example, the rule of weakening (*S*|-*C / S, A*|-*C*) states that if *C* can be validly derived from a sequence of formulas *S*, then *C* can be derived from *S* plus an additional formula *A*—this is known as the property of monotonicity. Relevance logicians have argued that the random adjunction of premises is not permissible, because the added premise may not be relevant to the conclusion. On the relevance logician’s narrower construal of logical consequence, an application of weakening may thus transform a valid argument into an invalid one. Relevance logics are thus examples of substructural logics.

Another example of a structural rule is the rule of contraction according to which two occurrences of the same formula type can be contracted into a single one: *S, A, A*|-*C* / *S, A*|-*C*. The system of *linear logic* is one that disallows the rule of contraction. Linear logics model resource-conscious deductive processes, i.e. processes for which it matters how many times a premise type is being appealed to in the course of a deduction, perhaps because repeated use is costly some sense. Notice that the assumption that *S* is a sequence rather than a set is crucial in this example and the next.

A final example is the rule of commutativity according to which the order of the premises can be freely altered: *S, A, B*|-*C* / *S, B, A*|-*C*. This structural rule too may be restricted. For instance, it does not hold in the *Lambek calculus* that aims to model grammatical categories and hence is order-sensitive.

See also philosophy of logic, proof theory, sequent calculus.